Price Competition with LTE-U and WiFi

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***Abstract*—LTE-U is an extension of the Long Term Evolution (LTE) standard for operation in unlicensed spectrum. LTE-U differs from WiFi, the predominant technology used in unlicensed spectrum in that it utilizes a duty cycle mode for accessing the spectrum and allows for a more seamless integration with LTE deployments in licensed spectrum. There have been a number of technical studies on the co-existence of LTE-U and WiFi in unlicensed spectrum In this paper, we instead investigate the impact of such a technology from an economic perspective. We consider a model in which an incumbent service provider (SP) deploys a duty cycle-based technology like LTE-U in an unlicensed band along with operating in a licensed band and competes with one or more entrants that only operate in the unlicensed band using a different technology like WiFi. We characterize the impact of a technology like LTE-U on the market outcome and show that the welfare impacts of this technology are subtle, depending in part on the amount of unlicensed spectrum and number of entrants. We also investigate the impact of the duty cycle and the portion of unlicensed spectrum used by the technology.**

I. Introduction

Offloading traffic to unlicensed spectrum has been a vital approach for wireless service providers (SPs) to meet the ever rising demand for mobile data and retain control over profit margins [1]–[3]. Indeed, in 2016, there was more mobile data traffic offloaded to unlicensed bands than served in licensed spectrum worldwide [4]. These trends have led to the development of technologies for unlicensed access that are based on the LTE technology that SPs utilize in licensed spectrum. The two main examples of this are LTE in unlicensed spectrum (LTE-U) and License Assisted Access (LAA). These differ in several ways from the WiFi technologies that are widely used in the same unlicensed spectrum. For example, both LTE-U and LAA utilize LTE's carrier aggregation capability to essentially combine a SPs licensed and unlicensed spectrum. Moreover, LTE-U differs in that it does not employ a listen-before-talk (LBT) protocol as used by WiFi, but instead is based on a duty-cycle based approach.[1] This led to much interest in studying the co-existence of WiFi and LTE-U from a technical point of view, e.g. [6]–[9]. In this paper, we instead examine the impact of such technology from market point-of-view. Namely, we seek to understand the impact of a SP using a technology like LTE-U on the competition with other SPs that utilize a technology like WiFi.

We consider a scenario similar to that in [10], [11], where SPs compete for customers by announcing prices for their service (see also [12]–[14]). The customers select SPs based on the sum of the price they pay for service and a congestion cost that is incurred for using the given band of spectrum. In [10], [11], the SPs compete by announcing one price for service in an unlicensed band and a different price for service in any licensed band which the SP may own. When LTE-U technology is adopted, we instead assume the SP can announce a single price because of the seamless integration of LTE technology on both the licensed and unlicensed bands. In our model, the duty cycle mechanism of LTE-U is considered while other aspects of LTE-U are ignored.[2] Under the duty cycle setting, the incumbent SP (with licensed spectrum) is able to use both its licensed spectrum and a portion of the unlicensed spectrum to serve customers when the duty cycle is 'ON' while it can only use its own licensed band when the duty cycle is 'OFF'. In such a scenario, we model customers as being sensitive to the average congestion they experience across the whole duty cycle.

We use α and β to denote the duty cycle and the portion of unlicensed spectrum that are used for LTE-U, respectively. We first consider α and β as fixed parameters determined by a regulator. For example, currently LTE-U channel bandwidth is set to 20 MHz which corresponds to the smallest channel width in WiFi and Qualcomm recommends that LTE-U should use a period of 40, 80 or 160 ms, and limits maximal duty cycle to 50% [15]. We show that when there are multiple entrant SPs, adopting LTE-U technology can help the incumbent SP to increase revenue and also benefit social welfare when the bandwidth of unlicensed spectrum is small. When there is only one entrant SP in the market, we show that it is possible for LTE-U technology to hurt the revenue of the incumbent. Then we consider α as a controllable parameter with fixed β. We show that with multiple entrant competitors, the incumbent's revenue increases with α while with one entrant, the incumbent may prefer a small α. Finally, we consider varying α and β while keeping the utilization of unlicensed spectrum ($\alpha\beta$) constant. We show that when the unlicensed bandwidth is small, the incumbent may prefer lower α and higher β. But when the unlicensed bandwidth is large, the incumbent may prefer a higher α and a lower β.

In terms of other related work, [16] also considers an economic model of LTE-U and WiFi. In [16] the focus is not on competition between LTE-U and WiFi providers (there is only one licensed service provider) but rather on understanding how LTE-U impacts the service selection of a finite number of

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[1]LAA does employ LBT, which is required in some parts of the world. LTE-U was developed first and is being deployed in countries where LBT is not required for unlicensed channel access. For example, T-Mobile launched LTE-U in spring 2017 [5] to serve customers in many cities in the US.

[2]For example, LTE and WiFi will also differ in spectral efficiency, which we do not capture here, but leave for future work. Our reason is to focus on the different MAC layer schemes of these two approaches.

users, each with a "congestion tolerance" for the service they select. In this work, WiFi is a free option that is congestible, while the licensed service is not congestible but is available at a cost.

The rest of the paper is organized as follows. Our model is described in Section II. We first consider the case without competition in Section III. Then we treat α and β as fixed parameters in Section IV and compare the results with the monopoly case and the model in [10], [11]. In Section V, we view α and β as controllable variables and investigate their impact on the incumbent SP. Some numerical results are shown in Section VI. Finally, we conclude in Section VII. Several proofs are omitted due to space considerations.

II. System model

We consider a market with one incumbent SP and N entrant SPs, where the incumbent SP uses a duty-cycle based technology such as LTE-U. In the following we will simply refer to this as LTE-U, though as noted previously this is not intended to model every aspect of LTE-U. The incumbent SP is assumed to possess its own licensed band of spectrum with bandwidth B, while entrants have no licensed spectrum. There is a single unlicensed band with bandwidth W that can be used by both the incumbent and entrant SPs. When the incumbent SP applies the LTE-U technology, it uses carrier aggregation on the unlicensed band and operates in a duty cycle mode. When LTE-U is in 'ON' mode, we assume that it is always using a portion of the unlicensed spectrum so that entrant SPs are not able to operate over this portion (e.g. due to LBT, the entrants would sense the incumbents presence and not transmit). We use α to denote the percentage of time that the SP aggregates the unlicensed spectrum. We use β to denote percentage of unlicensed spectrum that the incumbent uses when duty cycle is in 'ON' state, i.e., when the LTE-U is 'ON', the bandwidth that the incumbent can use becomes $B + \beta W$.

The SPs are assumed to compete for a common pool of infinitesimal customers by setting prices for their services. Without loss of generality, we assume that the incumbent is SP 1, and all the entrant SPs are indexed from 2 to $N + 1$. The price announced by SP i is denoted by p_i. The SPs serve all customers that accept their price. The revenue of SP i is then $x_i p_i$, where x_i is the customer mass that accept price p_i.

As in [12]–[14], a SP's service is characterized by a congestion cost. The congestion that the customers experience in a band is denoted by $g(X, Y)$, which is assumed to be increasing in the total customer mass X on the band and decreasing in the service bandwidth Y. Here, we assume a specific form $g(\frac{X}{Y})$, where $g(\cdot)$ is a convex increasing function with $g(0) = 0$ and $\frac{X}{Y}$ is the number of users per unit bandwidth in a given band. When the incumbent SP applies LTE-U technology, the congestion that the customers experience will vary across the duty cycle. We assume that customers are sensitive to the average congestion across the duty cycle.[3] The average congestion of customers served by the incumbent SP is then given by

$$\hat{g}_{in}(x_1) = \alpha g(\frac{x_1}{B + \beta W}) + (1 - \alpha) g\left(\frac{x_1}{B}\right).$$

[3]This is reasonable as over the time-scale that customer select SPs they will receive service over many duty cycles.

The average congestion experienced by customers who choose an entrant SPs is

$$\hat{g}_{en}(\mathbf{x}) = \alpha g\left(\frac{\sum_{j=2}^{N+1} x_j}{(1-\beta)W}\right) + (1 - \alpha) g\left(\frac{\sum_{j=2}^{N+1} x_j}{W}\right).$$

Note that as in [10] [11], the congestion experienced in the unlicensed band by a customer of an entrant SP is the same for all entrants and depends on the total traffic across all entrants. This is modeling the fact that the entrants are all employing a technology like WiFi to share this band. Also note that we assume that when the LTE-U duty cycle is on, the entrant can only use the remaining $(1 - \beta)W$ of the spectrum.

As in [10], [11], we assume that customers seek to receive service from the SP with the lowest *delivered price*, which is given by the sum of the announced price and the average congestion cost of that SP's service. This captures the fact that customers are sensitive both to cost of service and the quality of service. Hence, for the incumbent SP, the delivered price $d_1(p_1, \mathbf{x})$ is denoted by $p_1 + \hat{g}_{in}(x_1)$. For an entrant SP, its delivered price $d_i(p_i, \mathbf{x}), i \geq 2$ is given by $p_i + \hat{g}_{en}(\mathbf{x})$.

We assume that customers are characterized by an inverse demand function $P(q)$, which indicates the delivered price at which a mass of q customers are willing to pay for service. We assume $P(q)$ is concave decreasing. Each customer is infinitesimal so that a single customer has a negligible effect on the congestion in any band. Therefore, given the announced price by the SPs, the demand of service for each SP i is assumed to satisfy the Wardrop equilibrium conditions [17]. In our model, the conditions for the SPs are

$$d_i(p_i, \mathbf{x}) = P\left(\sum_{j=1}^{N+1} x_j\right), \text{for } x_i > 0,$$

$$d_i(p_i, \mathbf{x}) \geq P\left(\sum_{j=1}^{N+1} x_j\right), \text{for } \forall i,$$

where $d_i(p_i, \mathbf{x})$ is the delivered price of SP i. The conditions imply that at the Wardrop equilibrium, all the SPs serving a positive amount of customers will end up with the same delivered price, which is given by the inverse demand function. A Nash equilibrium of the game is one in which the customers are in a Wardrop equilibrium and no SP can improve their revenue by changing their announced price.

At an equilibrium, the customer surplus is defined as the difference between the delivered price each customer pays and the amount it is willing to pay, integrated over all the customers, i.e.,

$$CS = \int_0^Q P(q) - P(Q) dq, \tag{1}$$

where $Q = \sum_j x_j$. The social welfare of the market is the sum of consumer welfare and the SPs' revenue:

$$SW = CS + \sum_j p_j x_j. \tag{2}$$

III. Monopoly scenario

Before analyzing a competitive setting, we first examine a scenario in which there is only a single incumbent and no entrants. Hence, the incumbent is a monopolist and can use both the licensed and unlicensed band. Our goal in this section is to show that for our LTE-U model, such a monopolist would have no incentive to deploy the new technology.[4] This shows that in later sections when the incumbent does deploy such a technology that it is due to competitive factors and not an inherent advantage of the technology.

In this section, we allow the incumbent to offer both service using LTE-U (with a given α and β) and additionally an "unlicensed service" that uses the remainder of the unlicensed band when the LTE-U duty cycle is off.[5] This ensures that using LTE-U does not reduce the amount of spectrum the incumbent has access to. The incumbent's revenue optimization is then given by:

$$\begin{aligned} &\max_{p_1^l, p_1^u} \quad p_1^l x_1^l + p_1^u x_1^u \qquad (3)\\ \text{s.t.} \quad & p_1^l + \alpha g\left(\frac{x_1^l}{B+\beta W}\right) + (1-\alpha) g\left(\frac{x_1^l}{B}\right) = P(x_1^l + x_1^u),\\ & p_1^u + \alpha g\left(\frac{x_1^u}{(1-\beta)W}\right) + (1-\alpha) g\left(\frac{x_1^u}{W}\right) = P(x_1^l + x_1^u),\\ & p_1^l, p_1^u \geq 0. \end{aligned}$$

Here, p_1^l is the price the incumbent offers for serving x_1^l customers using LTE-U; p_1^u and x_1^u are the corresponding values for the unlicensed service. The first two constraints enforce the Wardrop equilibrium conditions for these two services. Note also that if we set $\alpha = 0$ and $\beta = 0$, then this reduces to a model as in [10] in which the incumbent does not employ LTE-U and offers separate licensed and unlicensed service.

Consider the expected congestion for the LTE-U service in (3). Given the convexity of the congestion function $g(\cdot)$, we have the following inequality:

$$\begin{aligned} \hat{g}_{in}(x_1^l) &\geq g\left(\frac{\alpha x_1^l}{B+\beta W} + (1-\alpha)\frac{x_1^l}{B}\right)\\ &\triangleq g\left(\frac{x_1^l}{B_e}\right), \end{aligned}$$

where B_e denotes the *equivalent licensed bandwidth* given by

$$B_e = B + \frac{\alpha\beta W}{1 + \beta(1-\alpha)W/B}. \qquad (4)$$

Similarly, considering the congestion for the unlicensed service, we have the *equivalent unlicensed bandwidth* W_e given by

$$W_e = W - \frac{\alpha\beta W}{1 - \beta(1-\alpha)}. \qquad (5)$$

Note that the congestion is no smaller than in a setting where the incumbent offered separate licensed and unlicensed services (without LTE-U) using the equivalent bandwidth and equality holds when the congestion function $g(\cdot)$ is linear. Based on this, we have the following result for the monopoly case.

Theorem 1: In a monopoly scenario, the incumbent SP can gain no additional revenue by using the LTE-U technology.

The detailed proof is shown in Appendix A. Notice that the equivalent licensed bandwidth increases and the equivalent unlicensed band decreases but the total amount of equivalent bandwidth in (4) and (5) decreases when adopting LTE-U. This is the reason that prevents the incumbent from adopting LTE-U.

IV. Competition with Fixed α and β

We now study the case where there is competition between the incumbent and one or more entrants. We first consider the case where there are multiple entrants and then consider the special case of one entrant. In both cases we will see that unlike the previous section, the incumbent may not have an incentive to deploy LTE-U. Throughout this section we assume that α and β are fixed.

A. One incumbent & multiple entrants

In this section, we assume that there $N \geq 2$ entrants. Hence, these entrant will compete with each other and not just with the incumbent. The presence of this competition yields the following result on the entrants' equilibrium prices.

Lemma 1: If there are at least two entrant SPs in the market, in any Nash equilibrium every entrant SP i serving a positive mass of customers must have $p_i = 0$ and at least two SPs must announce this price.

Lemma 1 is similar to a result in [10] where firms compete in unlicensed spectrum without LTE-U. Essentially, since the entrant SPs are all offering the same service due to sharing the same spectrum, they will be incentivized to compete the price for this service to zero. Hence, all the entrant SPs get no revenue regardless of the incumbents actions.[6] A corollary of this result is that the incumbent would have no incentive to offer a separate unlicensed service in this setting as its price for this would also be zero. Based on Lemma 1 we give the following result on the incumbent's revenue.

Theorem 2: Consider one incumbent and multiple entrants. Given a fixed $\alpha > 0$ and $\beta > 0$, the following hold:

1) The incumbent SP announces a higher price and attracts more customers when LTE-U is adopted. As a result the incumbent SP gets a higher revenue.
2) The customer mass served by the entrant SPs decreases when the incumbent SP uses LTE-U technology.
3) The total customer mass served by the incumbent and entrant SPs is lower when LTE-U is adopted.

As in the previous section, the use of LTE-U increases the equivalent licensed bandwidth. However, now that there are multiple entrants keeping the price on the unlicensed band

[4] Again, we emphasize that here we are focusing on the duty-cycle MAC protocol and not other aspects of LTE-U. For example, if our LTE-U model included a gain in spectral efficiency, then a monopolist might gain by deploying it.

[5] In subsequent sections, the incumbent will only offer service using LTE-U or unlicensed service, instead of this combination.

[6] Note here we are ignoring any cost for offering service. If such a cost was included the result would be that the price is competed down to cost, sill yielding zero profit.

zero, this benefit to the incumbent is not offset with a loss due to the reduced equivalent unlicensed bandwidth. Due to the improved service by adopting LTE-U, the incumbent is able to announce a higher price and at the same time attract more customers. That will lead to an increase in the revenue. However the delivered price will increase, resulting in fewer customers served.

Theorem 3: Customer surplus with LTE-U is nondecreasing as the amount of unlicensed bandwidth W increases but is always less than the consumer surplus achieved without LTE-U.

Theorem 3 shows that adding more unlicensed spectrum benefits customers while adopting LTE-U always hurts customers when there are multiple entrant SPs in the market. This is because increasing the bandwidth of unlicensed spectrum actually increases the amount of resources for both the incumbent and entrant SPs. But when LTE-U is used, there is some loss in the total equivalent bandwidth and the incumbent is able to increase its price, leading to a loss in customer surplus.

So far we have seen that with multiple entrants, LTE-U increases firm profits (namely those of the incumbent) but decreases consumer welfare. We next consider the impact of this technology on the overall welfare which includes both of these factors. We first use a simplified example to gain insight and then give a more general result.

Consider a homogeneous inverse demand function and linear congestion cost, i.e., $P(q) = T$ if $q \leq A$, $P(q) = 0$, otherwise, and $g(x) = x$. Here, A can be viewed as the size of the market while T indicates the valuation of every consumer. We then have the following result.

Theorem 4: With a homogeneous inverse demand function and linear congestion, if $W \leq \frac{\sqrt{A^2+B^2T^2-BT+A}}{2T}$, social welfare will always increase when LTE-U is adopted for any $\alpha, \beta > 0$. Otherwise, social welfare can either increase or decrease when LTE-U is adopted.

The proof is shown in Appendix B. Theorem 4 shows that under the assumption of a homogeneous inverse demand function and linear congestion, when the unlicensed bandwidth W is small, adopting LTE-U is good for social welfare no matter what α and β are. Note that in these cases, customer surplus is always zero and all the social welfare comes from the revenue of the incumbent SP. When W is beyond the threshold, it becomes unclear how social welfare changes with LTE-U. It can depend on the choice of α and β. Also note that the threshold bandwidth is an increasing function of $\frac{A}{T}$, which is the ratio between market size and customer valuation. When there are more customers in the market or the customers valuation goes down, the threshold goes up, which means it is more likely that adopting LTE-U increases welfare. This is because when there are more customers, or the customers have a lower valuation, the incumbent is incentivized to serve more customers to increase its revenue.

The following results extends Theorem 4 to a general inverse demand function and a general congestion function.

Theorem 5: Given a fixed $B > 0$, $\alpha > 0$ and $\beta > 0$, There exists some $W_{th} > 0$ such that when $W < W_{th}$ adopting LTE-U achieves a higher social welfare than that without LTE-U. When $W \geq W_{th}$, the social welfare can be either better or worse with LTE-U. But when W is large enough LTE-U always hurts social welfare.

A linear approximation method as in [18] is used to prove the theorem. When LTE-U is adopted, Theorem 3 shows customer surplus decreases, which means the delivered price should increase. When the bandwidth of unlicensed spectrum is small, the increase in revenue of the incumbent is able to compensate for the customer surplus loss, so that the overall social welfare can increase. But when W is large, the advantage of LTE-U may not be large enough to raise the delivered price to make up for the loss of customer surplus, which will result in a loss of social welfare.

B. One incumbent & one entrant

We next consider the case with only one entrant. If only this entrant is offering unlicensed service, then Lemma 1 no longer applies and so this case requires a separate analysis. Before considering the impact of LTE-U, we first consider two possible ways the incumbent SP could act without this technology: it could compete with the entrant to serve customers on the unlicensed band or it could only serve customers on the licensed band. We call the first case *unlicensed sharing* and in this case, the results are the same as when an incumbent without LTE-U competes with multiple entrants. We call the second case *licensed sharing*; in this case, the entrant SP is able to use the unlicensed spectrum exclusively. The objective of each SP is still to maximize revenue while the Wardrop equilibrium conditions are satisfied. To be precise, in the licensed sharing case, the conditions for the entrant SP on the new band become

$$\begin{aligned} p_2 + g\left(\tfrac{x_2}{W}\right) &= P\left(x_1 + x_2\right), \text{if } \ x_2 > 0 \\ p_2 + g\left(\tfrac{x_2}{W}\right) &\geq P\left(x_1 + x_2\right), \text{otherwise.} \end{aligned}$$

We first give a brief result to compare the licensed sharing and unlicensed sharing case without LTE-U.

Lemma 2: In the case with one incumbent and one entrant SP, both the incumbent and entrant SPs are able to gain higher revenue with licensed sharing than with unlicensed sharing.

Lemma 2 shows that rather than making the spectrum unlicensed, both the incumbent and entrant would prefer that it is exclusively licensed to the entrant SP. However, note that if the incumbent has the option of unlicensed sharing, then this will not be an equilibrium as it would always want to enter the unlicensed market and capture some of the entrant's revenue.

Next we study of impact of LTE-U and in particular compare this to the licensed sharing case (which as noted above gives an upper bound on the incumbent's revenue in the unlicensed sharing case). In this subsection, we assume a linear congestion function $g(x) = x$ and inverse demand function $P(x) = 1 - x$ to simplify the calculations and give some insights.

Theorem 6: With a linear congestion cost and inverse demand, we have the following comparisons with licensed sharing:

1) When $\frac{B}{1-\alpha} < \frac{4}{3}$, the incumbent SP can always gain higher revenue with LTE-U. Otherwise, the incumbent can be

either better or worse off with LTE-U (depending on the parameter values).

2) For any $\alpha, \beta \in (0, 1)$, there always exists some W_{th}, such that when $W < W_{th}$, the incumbent SP can gain higher revenue with LTE-U.

Both statements in Theorem 6 give sufficient conditions to guarantee a larger revenue for the incumbent SP with LTE-U. Equation (4) and (5) show that LTU-U increases the amount resources of the incumbent and at the same time reduces the amount of resources of the entrant. Intuitively, this should lead to higher revenue for the incumbent with LTE-U. The first statement in Theorem 6 shows that this intuition holds when the incumbent's licensed spectrum is sufficiently small. However, when there is a large enough amount of licensed spectrum, the incumbent SP may suffer a loss in revenue with LTE-U. This is because the incumbent can already serve a large amount of customers on the licensed band and reducing the entrant's resources causes it to reduce the delivered price, lowering the incumbent's revenue. The second statement of Theorem 6 claims that as long as there is not too much unlicensed spectrum, the incumbent is always willing to adopt LTE-U, which yields a higher revenue. That is because, when W is relatively small, using LTE-U can decrease the equivalent bandwidth of the entrant competitor, which increases the congestion on unlicensed band significantly, giving an advantage to the incumbent SP. But when W is large, the decrease of the entrants' spectrum resource does not have a significant impact on congestion. As a result LTE-U can not increase the customer mass served by the incumbent enough to compensate for the lowered price due to competition. So the incumbent may not want to use LTE-U.

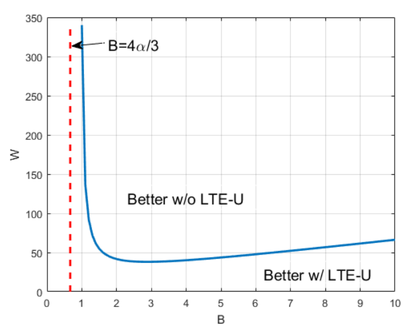


Fig. 1: The bandwidth regions where LTE-U is better and worse with $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$.

We use Fig. 1 to illustrate the region where the incumbent SP can get more revenue with LTE-U. We choose $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$ in the figure. When B and W lie below the blue curve, the incumbent is better off with LTE-U. The red dashed line represents $B = \frac{4\alpha}{3}$. The blue curve approaches to the red line asymptotically when $W \to \infty$. We can also see that the unlicensed bandwidth threshold W_{th} is relatively large compared to the threshold for licensed bandwidth (the red dashed line). That means in most practical cases, the incumbent SP would be willing to use LTE-U technology.

Next we characterize the customer surplus in the case of one incumbent one entrant SP.

Theorem 7: When there is one incumbent and one entrant SP, for any value of B, α and β, customer surplus is non-decreasing with unlicensed bandwidth W. Also, there exists some $W_{th} \geq B$, such that when $W \leq W_{th}$, customer surplus decreases when LTE-U is adopted compared to licensed sharing and when $W > W_{th}$, customer surplus increases.

The first result in Theorem 7 is consistent with that in the multiple entrants case, which shows customer surplus increases with the bandwidth of unlicensed spectrum. The second result is a slightly different, which shows that LTE-U can hurt customer surplus when W is relatively small but it is able to improve customer surplus when W is large, while in the multiple entrants case, customer surplus always becomes worse with LTE-U. The reason is that when there is only one entrant SP, the price on the unlicensed band is not zero, which means a certain amount of consumers in the market are not served. When W is large and LTE-U is used, the incumbent can use the additional unlicensed spectrum to alleviate congestion without hurting the entrant SP too much. As a result, more consumers in the market can be served and customer surplus increases. Also note that when W is relatively small, the loss in consumer surplus is balanced out by the increase in the SP's revenue.

We next examine how social welfare changes when LTE-U is adopted in the asymptotic case with $W \to \infty$.

Theorem 8: In the case with one incumbent and one entrant SP, if $W \to \infty$, social welfare always increases when LTE-U is adopted.

Theorem 8 shows that in the single entrant case, social welfare is higher with LTE-U when there is a large amount of unlicensed spectrum. Recall that in Theorem 5, we show in the case with multiple entrant SPs, social welfare is better with LTE-U when the bandwidth of unlicensed spectrum is small. The intuition is that with multiple entrants, the price is competed to 0 on the unlicensed band. As a result, the increment in revenue can only cover the loss of customer surplus when a small mass of customers are served in the market, which implies a small W. However, in the single entrant case, the price is not zero, there can be larger amount of customers unserved in the market, which leaves enough room for customer surplus to improve. When W is large, the improvement of customer surplus can make up for the loss of revenue by the SPs.

V. Impact of α and β

The duty cycle, α, and the percentage of the band for LTE-U use, β, are two important parameters to maintain fair and efficient coexistence of LTE-U and other unlicensed spectrum users. In this section, we investigate the impact of α and β in both cases with multiple entrants and one entrant. To give a better understanding of how α and β influence the revenue and welfare, we again consider a linear model where the inverse demand function is $P(x) = 1 - x$ and the congestion function is $g(x) = x$.

A. Impact of duty cycle

First we consider the case that β is fixed and only vary the duty cycle α to see its impact. We begin with the case of one incumbent and multiple entrant SPs in the market. The following proposition characterizes the Nash equilibrium in this case.

Proposition 1: Assuming a linear inverse demand function and congestion function and multiple entrants, the equilibrium announced price of the incumbent SP and the customer mass served is

$$p_1 = \frac{1}{2(1+W_e)}, \quad x_1 = \frac{B_e}{2(1+B_e+W_e)},$$

where B_e and W_e are defined in (4) and (5) respectively. The announced prices of entrant SPs are all zero and the total customer mass served by the entrants is

$$w_t = \frac{W_e(2+2W_e+B_e)}{2(1+W_e)(1+B_e+W_e)}.$$

From this proposition, we can see that the equilibrium price can be expressed with the equivalent bandwidth in (4) and (5). We next investigate how the equivalent bandwidth B_e, W_e and their sum change with the duty cycle α.

Lemma 3: The equivalent bandwidth B_e increases with α and W_e decreases with α. If $W > \frac{B}{1-\beta}$, for $\alpha \in (0, \frac{1}{2})$, the total amount of equivalent bandwidth $B_e + W_e$ always decreases with α.

This lemma shows that when B is relatively smaller than W, the total equivalent bandwidth decreases with α in the range $(0, \frac{1}{2})$. As mentioned previously, the duty cycle is usually limited below 50%. That implies that in practice, the total equivalent bandwidth decreases with α.

Theorem 9: When there is one incumbent SP and multiple entrant SPs, the revenue of the incumbent always increases with duty cycle α.

Theorem 9 is a natural result of Lemma 3. Because the incumbent SP gets more equivalent bandwidth with LTE-U while the entrants lose more resources with increasing α, the incumbent's revenue should increase with α. Consequently, if there is no limit for choosing α, the incumbent SP may want to raise α to a value close to 1.

Things become different when we consider the case with only one entrant SP in the market. In this case, Lemma 3 still holds, but the incumbent SP may not want to choose a large α all the time. The following theorem describes such an example.

Theorem 10: When there is only one incumbent and one entrant in the market and $W \to \infty$, the optimal α for the incumbent SP to maximize its revenue is $\alpha^* = \max\{1 - \frac{3B}{4}, 0\}$.

Theorem 10 shows that the revenue of incumbent is no longer increasing with α when there is only one entrant SP in the market. Fig. 2 shows how revenue changes with α when $B = 1$, $W \to \infty$ and $\beta = 0.2$. We can see the revenue of the incumbent reaches a maximum when $\alpha = \frac{1}{4}$ and is higher than that without LTE-U. In the case with one entrant SP, the incumbent SP may want to choose a small α or even does not want to use LTE-U technology ($\alpha^* = 0$) when there is plenty of licensed resource. Another thing to notice is that when $W \to \infty$, the optimal α is non-increasing with licensed bandwidth B. This implies the more licensed spectrum the SP possesses, the smaller duty cycle it may prefer.

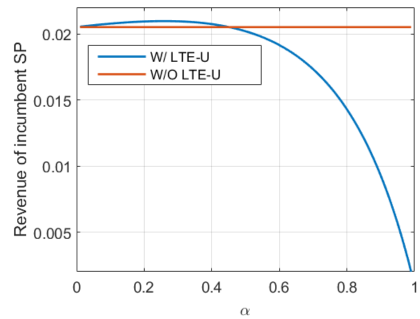


Fig. 2: Revenue of the incumbent in the case with one entrant SP when $B = 1$, $W \to \infty$ and $\beta = 0.2$.

Next we look at the social welfare. Theorem 8 states that in the case with one incumbent, one entrant and $W \to \infty$, LTE-U yields higher social welfare. We characterize the gap between the two cases in the following theorem.

Theorem 11: When there is one incumbent and one entrant SP, if $W \to \infty$, the social welfare gap between the cases with and without LTE-U is non-decreasing in α.

Theorem 11 shows that when there is a sufficient amount of unlicensed spectrum, a regulator may prefer a larger duty cycle α, because it virtually increases the total amount of resources in the market, which results in a larger increase in social welfare.

B. Fixed utilization ratio

In this section we fix the incumbent's utilization of the unlicensed spectrum, given by the product $\alpha\beta$. We then study the impact of varying α and β keeping this product fixed for the case of one incumbent and multiple entrants. We set $\alpha\beta = k$, where k is a constant. We then view α as a variable in the analysis. In this case, α can vary in the range $(k, 1)$. The equivalent bandwidth can be rewritten as

$$B_e = B + \frac{kW}{1 + (k/\alpha - k)W/B}, \; W_e = W - \frac{kW}{1-(k/\alpha - k)}. \tag{6}$$

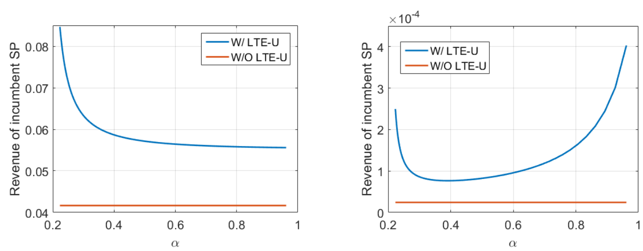
Note that both B_e and W_e are increasing in α. As a result, the total amount of equivalent bandwidth increases with α. But it remains unclear what impact this has on the incumbent's revenue. The following theorem addresses this.

Theorem 12: In the case with one incumbent SP and multiple entrant SPs under the linear setting, if $\alpha\beta = k$ and k is a constant in $(0, 1)$, then:

1) If $B > \frac{\sqrt{2}}{2}$ and $W \leq B$, incumbent's revenue always decreases with α in the range $(k, 1)$;
2) For any choice B, there always exists some $W_{th} > 0$ and $k_{th} \in (k, 1)$, such that when $W > W_{th}$, the incumbent's revenue decreases with β in the range (k, k_{th}).

Theorem 12 shows that in different situations, the incumbent SP has different preference on higher α or β when the product $\alpha\beta$ is fixed. When the bandwidth of the unlicensed spectrum is relatively small, the amount of equivalent bandwidth increases with α, but the revenue decreases with α. In this case, using a larger portion of unlicensed spectrum is more profitable than using a small portion for a longer time. However when W is relatively large, the incumbent's revenue decreases with β in some range, which implies that the incumbent may prefer a larger α. In this case, a small portion of the spectrum may be enough for the incumbent to serve its customers. As a result a larger duty cycle α might be more profitable for the incumbent SP.

Fig. 3 is an example of these two cases. We fix $k = 0.2$ and $B = 1$. We can see in Fig. 3(a), when W is relatively small, the incumbent may prefer a lower α. But when W is relatively large, the incumbent may prefer a higher α as is shown in Fig. 3(b).



(a) $B = 1, W = 1$ (b) $B = 1, W = 100$

Fig. 3: Revenue of incumbent SP with multiple entrant SPs and $k = 0.2$.

Theorem 13: In the case with one incumbent SP and multiple entrant SPs in the market, if $\alpha\beta = k$ and k is a constant in $(0, 1)$, then customer surplus always increases with α in (k,1).

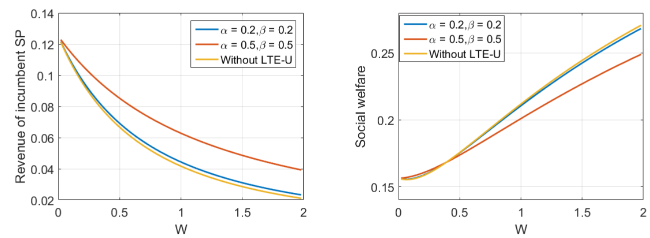
From the expression of equivalent bandwidth in (6), we know both B_e and W_e are increasing with α, which implies the total amount of equivalent spectrum resources increases with α. So in this case, a higher α can help increase the amount of virtual resources and serve more customers.

VI. Numerical Results

In this section we give some additional numerical examples illustrating our results. We again consider a model with a linear inverse demand function and congestion function where $P(x) = 1 - x$, $g(x) = x$. Both cases with fixed α, β and varying α, β are considered.

A. Fixed α and β

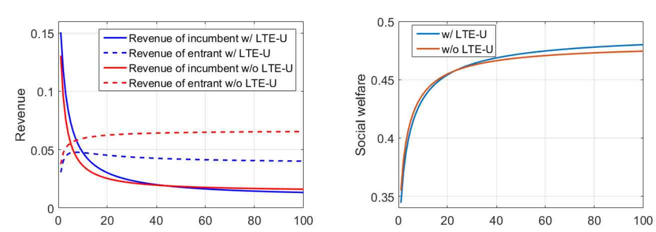
First we examine how the incumbent's revenue and social welfare changes with the amount of unlicensed spectrum when there is one incumbent SP and multiple entrant SPs in the market. We fixed the licensed bandwidth as $B = 1$ and set α, β to different values. The results are shown in Fig. 4. As is described in Theorem 2, the incumbent is always gaining more revenue with LTE-U as shown in Fig 4(a). Also we can see that when more spectrum can be used by LTE-U, and a higher duty cycle is allowed, the revenue is higher. The resulting social welfare is show in Fig. 4(b). We can see that when the bandwidth of additional unlicensed spectrum is small, social welfare increases with the adoption of LTE-U technology. But when more unlicensed spectrum is released, social welfare can be hurt with LTE-U. Another thing to notice is that social welfare decreases with W when W is small. This effect is also mentioned in [10]. LTE-U technology is able to make the social welfare loss smaller.



(a) Revenue of incumbent (b) Social welfare

Fig. 4: Comparison of revenue and welfare in the case with multiple entrant SPs.

Next we take a look at the case with one incumbent and only one entrant SP in the market. We fix $B = 5$, $\alpha = 0.5$, $\beta = 0.5$. Results are shown in Fig. 5. We also include the entrant's revenue in Fig. 5(a) . We can see that when W is relatively small, the incumbent is able to gain more revenue with LTE-U while the entrant SP suffers a loss in revenue. However, when W is large, LTE-U hurts the revenue of both SPs. The results for social welfare are shown in Fig. 5(b). We can see when W is large, social welfare increases with LTE-U and there is a social welfare gap between the cases with and without LTE-U. Next, we let $W \to \infty$ and see how this gap changes with B under different α (β makes no difference when $W \to \infty$). Results are shown in Fig. 6. We can see that the social welfare gap first increases then decreases with B and always increases with α. When B is small, LTE-U is able to increase the amount of spectrum resources of the incumbent SP to serve more customers, which is good for the incumbent's revenue and customer surplus and as a result benefits the social welfare. However, when B is large, the gap is not as large because both cases approach the social optimal point so that the increase in resources cannot have as large an impact as when B is smaller.



(a) Revenue (b) Social welfare

Fig. 5: Comparison of revenue and welfare in the case with one entrant SP.

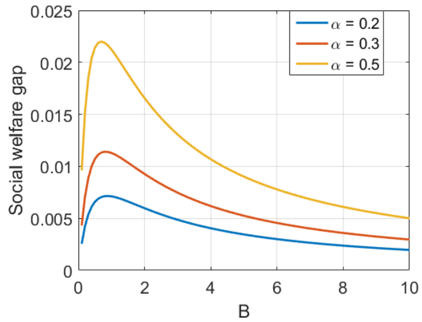
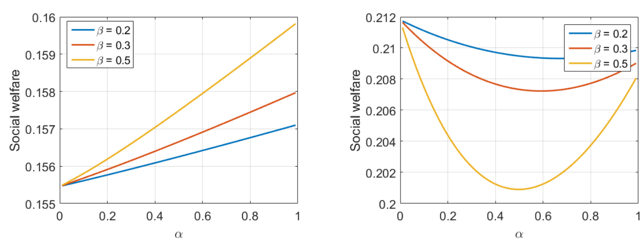


Fig. 6: Social welfare gap between the cases with and without LTE-U as a function of α and B.

B. *Varying α and β*

Next, we consider the impact of the duty cycle α with β fixed when there are multiple entrants in the market. We have already shown that the incumbent's revenue increases with α in Theorem 9. In Fig. 7, we show how social welfare changes with α for different values of licensed bandwidth, unlicensed bandwidth, and β. We can see that when W is small (Fig. 7(a)) , social welfare increase with the duty cycle α. In this case, a higher α is desirable by both the incumbent SP and the social planner. Also we can see that a larger β helps increase the social welfare. But when W is slightly larger (Fig. 7(b)), social welfare first decreases then increases with α. Additionally, we can see that when β increases, social welfare decreases.



(a) $B = 1, W = 0.1$ (b) $B = 1, W = 1$

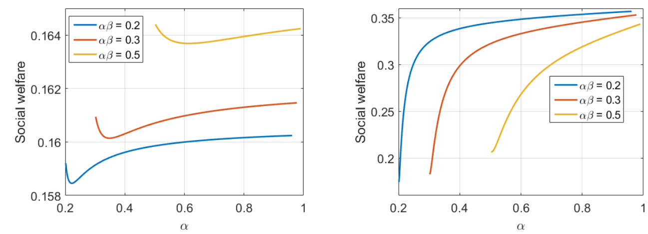
Fig. 7: Social welfare in the case with one entrant with fixed β.

Next we show how social welfare changes with α when $\alpha\beta$ is fixed in the case with multiple entrant SPs. Results are shown in Fig. 8. We can see in Fig. 8(a) that when W is relatively small, social welfare first decreases then increases with α. That is because there is some welfare loss when adding a small amount of unlicensed spectrum to the market as is described in [10]. Recall that the equivalent bandwidth of unlicensed spectrum increases with α when $\alpha\beta$ is fixed. As a result the social welfare may suffer when α increases in the case of small W. But when W is large as in Fig. 8(b), social welfare always increases with α.

VII. Conclusion

In this paper, we analyzed the market impact of LTE-U technology on the competition among incumbent and entrant SPs with licensed and unlicensed spectrum. We first analyzed the case where the duty cycle, α, and the portion of unlicensed spectrum that can be used by the incumbent, β, are fixed. Our results show that when there are multiple entrant SPs competing on the unlicensed band, the incumbent SP can get more revenue by using LTE-U. However when there is only one entrant SP in the market, the incumbent's revenue may decrease when LTE-U is adopted. We also show that the welfare impact of LTE-U depends on the market parameters - in some cases it can lead to a gain and in others a loss. We investigated the impact of α and β on the market. Our results show that when there are multiple entrants and if β is fixed, the incumbent's revenue increases with α. But when there is only one entrant SP using unlicensed spectrum, the optimal α is not necessarily 1 and can even be 0. We also fixed the product $\alpha\beta$ to see whether the incumbent prefers a high α or a high β. Results show that when the unlicensed bandwidth is relatively small, the incumbent prefers high β and when the unlicensed bandwidth is relatively large, the incumbent may prefer high α.

There are many ways this work could be extended. We only considered the duty cycle operation mechanism in LTE-U. We did not account for the different spectral efficiency between LTE and other technologies like WiFi. Incorporating such features is a possible extension. Other extensions include considering the investment costs for a SP to upgrade to LTE-U, competition among multiple incumbents and different types of customers.



(a) $B = 1, W = 0.2$ (b) $B = 1, W = 5$

Fig. 8: Social welfare versus α in the case with multiple entrants and fixed $\alpha\beta$.

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APPENDIX A
PROOF OF THEOREM 1

Proof: First, we claim that when congestion on both bands decreases, the optimal revenue of the SP increases, because the SP can just announce the same price and attract more customers while keeping the potential to increase revenue by adjusting price. As a result we only need to show that when the congestion level meets the lower bound, i.e. with linear congestion cost, the incumbent SP can gain no higher revenue than that without LTE-U.

We use the equivalent licensed and unlicensed bandwidth to rewrite the optimization in (3).

$$\begin{aligned} \max_{p_1^l, p_1^u} \quad & p_1^l x_1^l + p_1^u x_1^u \\ \text{s.t.} \quad & p_1^l + g\left(\frac{x_1^l}{B_e}\right) = P(x_1^l + x_1^u), \\ & p_1^u + g\left(\frac{x_1^u}{W_e}\right) = P(x_1^l + x_1^u), \\ & p_1^l, p_1^u \geq 0. \end{aligned}$$

For the optimization problem above, we can use x_1^l and x_1^u instead of p_1^l and p_1^u as optimization variable to reach the same result.

From the first order conditions of the objective function over x_1^l and x_1^u, we can show $\frac{x_1^l}{B_e} = \frac{x_1^u}{W_e} = \frac{x_1^l + x_1^u}{B_e + W_e}$, which means the congestion level on the licensed and unlicensed band should be the same. Since we can verify $B_e + W_e \leq B + W$, the total customer mass served becomes less when LTE-U is applied. As a result, we conclude that the revenue of the incumbent SP decreases. ■

APPENDIX B
PROOF OF THEOREM 4

Proof: First we consider the case without LTE-U technology. In this case, results on welfare are summarized as follows [13]:

1) When $W \leq \max\{\frac{A}{T} - \frac{B}{2}, 0\}$, we have zero customer surplus and
$$SW = \frac{BT^2}{4}; \tag{7}$$

2) When $\max\{\frac{A}{T} - \frac{B}{2}, 0\} < W \leq \frac{\sqrt{A^2+B^2T^2}-BT+A}{2T}$, we have zero customer surplus and
$$SW = (A - WT)\left[T - \frac{(A - WT)}{B}\right]; \tag{8}$$

3) When $W > \frac{\sqrt{A^2+B^2T^2}-BT+A}{2T}$, we have positive customer surplus and
$$SW = AT - \frac{A^2(B + 4W)}{4W(B + W)}. \tag{9}$$

We claim that in the first two cases, the adoption of LTE-U increases the social welfare. Recall that with a linear congestion function, LTE-U technology increases B to B_e and decreases W to W_e, where B_e and W_e are defined in (4) and (5), respectively.

In the first case, when we change B to B_e and W to W_e, since $W - W_e > B_e - B > 0$, we always have $W_e \leq \max\{\frac{A}{T} - \frac{B_e}{2}, 0\}$. This implies that (B_e, W_e) still falls in the region of case 1), so that we can still use equation (7) to calculate the social welfare. Obviously, when B increases to B_e, social welfare also increases.

In the second case, we claim that when LTE-U is adopted, the equivalent unlicensed bandwidth W_e can never go beyond the boundary $\frac{\sqrt{A^2+B_e^2T^2}-B_eT+A}{2T}$. Consider the following function :
$$f(b) = \frac{\sqrt{A^2 + b^2T^2} - bT + A}{2T}.$$

Given that $f(B) > W$ and $W - W_e > B_e - B$, we have
$$\begin{aligned} f(B_e) &= f(B) + \int_B^{B_e} f'(b)db \\ &> f(B) + \int_B^{B_e} -1db = f(B) - (B_e - B) \\ &\geq W - (W - W_e) = W_e. \end{aligned}$$

So in this case, when LTE-U is adopted, B_e and W_e can only fall into case 1) and case 2). Since all of the social welfare functions increase with B, it suffices to show that when reducing W to W_e, social welfare is nondecreasing. When fixing B, it can be shown that social welfare in (8) is decreasing in W when $W \geq \frac{A}{T} - \frac{B_e}{2}$ and achieves maximum $\frac{BT^2}{4}$ when $W = \frac{A}{T} - \frac{B_e}{2}$. Consequently, when decreasing W to W_e, if it still falls in the range of case 2), social welfare increases. If it falls into the range of case 1, it then becomes a constant with respect to unlicensed bandwidth W. As a result social welfare is nondecreasing when decreasing W to W_e.

■