The Maximum Stable Broadcast Throughput for Wireless Line Networks with Network Coding and Topology Control

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Abstract—We consider broadcasting from a single source to multiple destinations in a linear wireless erasure network with feedback. The problem is to find the maximum stable throughput under different transmission policies with opportunistic network coding and forwarding. Given stochastically varying traffic, we assume that network coding decisions are based on the availability of queued packets. The network is clustered into groups of terminals and network coding is applied locally to packets within each group. This allows us to evaluate the effects of topology control on the maximum stable rate. For each transmission policy we derive the optimal cluster size. We show that network coding improves the stable rate over plain retransmissions, and the network coding gain significantly benefits from opportunistic network coding, forwarding and topology control, ranging from 33% to 410%, depending on the physical channel parameters in the numerical experiments.

I. INTRODUCTION

Network coding is known to improve the achievable rates over routing in multicast networks [1]. Much of the work on network coding has focused on models in which terminals always have packets available to code and transmit. If traffic is random, queues in the network may occasionally become empty. In such cases, network coding decisions should be made dynamically, based on the instantaneous queue contents.

For stochastically varying packet traffic, a basic objective is to find the maximum stable throughput, i.e. the arrival rates such that all packet queues in the network have finite delay. For routing it is known that the back-pressure algorithms optimize the achievable stable rates [2]. This approach can be incorporated with random network coding [3] for general network topologies.

Network coding also helps for single-hop models, in particular, for broadcast erasure channels with feedback. The throughput and delay gains of random network coding have been studied in [4], [5] for broadcast systems with saturated queues. The maximum stable rate has been derived in [6]–[8] with queue-based dynamic network coding under random traffic. However, as the number of destinations increases, the benefits of network coding over a single hop are reduced to the bound imposed by the worst erasure probability, which increases with the number of one-hop destinations.

In this paper, we consider a linear wireless erasure network with receiver feedback. We cluster terminals into groups such that network coding is applied locally to packets within each group. This reduces the worst erasure probability but raises the need for joint network coding and forwarding of packets to propagate within each cluster and between clusters.

While [9] studied the capacity of erasure networks with backlogged traffic, we analyze the stable rate under random packet traffic as a function of cluster size. As an alternative to ARQ-based retransmissions, network coding is applied only at the first terminal in each cluster. Then, we allow packets to propagate within the cluster and apply network coding opportunistically at the intermediate terminals. The associated overhead for network coding is also discussed. The comparison of stable rates reveals that the network coding gain increases with opportunistic forwarding and topology control (in the form of choosing optimal cluster sizes).

The rest of the paper is organized as follows. Section II introduces the network model. A base-line retransmission policy is discussed in Section III. This is followed in Section IV by the introduction of a simple network coding policy over a single hop. Then, we introduce two improvements by allowing network coding at the intermediate terminals in Section V and by incorporating opportunistic forwarding in Section VI. The overhead for each transmission policy is discussed in Section VII and the stable rate performance is compared in Section VIII. Finally, we draw conclusions in Section IX.

II. SYSTEM MODEL AND RETRANSMISSION POLICIES

We consider broadcasting from a single source in a linear wireless erasure network. We assume the source is at one end of the network and we label it terminal 0; the other terminals are then labeled by the number of hops they are away from terminal 0. Transmissions only go in the direction of increasing i to avoid cycles. Each terminal can transmit a packet to a terminal i hops away with erasure probability ϵ_i, where ϵ_i increases with i. We assume a synchronous slotted system in which each transmission takes one time slot. The network is divided into groups of N terminals such that the last terminal in each group broadcasts packets to all terminals in the successive group on behalf of the source. The system model is shown in Fig. 1.
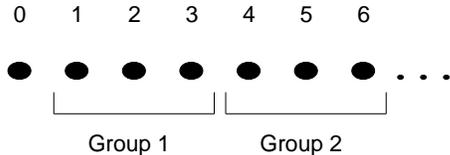
Fig. 1. A line network for $N = 3$. Terminals 0 and 3 are the group leaders of Groups 1 and 2 respectively. The erasure probability between terminals i and j, $j > i$, in the same group is ϵ_{j-i}.

The problem is to find the optimal group size N, where the optimality is defined based on the maximum stable arrival rate that the network can sustain under various transmission policies. For each group, we call the terminal preceding the group which broadcasts packets on behalf of the source the *group leader*, and call the terminal i hops away from the group leader *receiver* i. We assume the network operates on a group basis, *i.e.*, if a group is active, then either the group leader or a terminal in the group is transmitting; otherwise, the group leader and all terminals in the group are prohibited from transmissions.

We assume that whether a transmission is successfully received by each receiver is immediately known to all terminals within the group and to the group leader, possibly through a perfect feedback channel. The minimum transmission power is chosen to reach at most N-hop neighbor terminals, *i.e.*, both transmission and interference ranges are N hops. Therefore, every third group can be active, provided that there are at least three groups in the network.

For each transmission policy we consider, we will describe how the policy works within each group and calculate μ_N, the service rate for packets in *each group*. Therefore, *the arrival rate λ must satisfy the following stability condition:*[1]

$$\lambda < \frac{1}{3}\mu_N. \tag{1}$$

This fixed time-division policy achieves the same maximum rate as any other work-conserving policy, where (*a*) the system does not allow any idle slot provided that there are packets in the system, and (*b*) packet transmissions do not interfere. The probability that any group has a packet to transmit is given by the utilization factor of each group $\frac{\lambda}{\mu_N}$. Whenever one group is activated, then two other groups on both sides should stay idle to avoid interference. The stability condition is that the sum of utilization factors for separately activated groups is less than 1, which is equivalent to the stability condition (1).

In the following we propose four retransmission policies based on network coding and opportunistic forwarding. The first policy, simple network coding (SNC), is based on the fact that if every receiver in a group misses only one packet, then transmitting a linear combination from the group leader using these missing packets allows each receiver to receive the packet it misses. The second one, opportunistic network

coding (ONC), takes advantage that any receiver can transmit the linear combination of the missing packets if all its upstream receivers have received all packets. These two policies only consider packets that are missed at one receiver only. For those packets lost at at least two receivers, these policies simply assume these packets are lost at all receivers. To take care of these packets, we incorporate opportunistic forwarding to form the policies simple network coding with opportunistic forwarding (SNCOF) and opportunistic network coding with opportunistic forwarding (ONCOF), in which any receiver can take the role of the group leader for a packet if this packet is already received by all upstream receivers.

III. PLAIN RETRANSMISSION POLICY

Before we analyze the performance of these retransmission policies, we define a benchmark plain retransmission policy (PRP) based on standard ARQ. In PRP, the group leader transmits a new packet only when the previous packet has been received by all receivers. Otherwise, the same packet is retransmitted until all receivers have received it. This policy has been considered in [4], [6] in comparison with network coding. For a group of N terminals, the service rate is

$$\mu_{N,\text{PRP}} = \frac{1}{1 + \sum_{l=1}^{\infty}\left[1 - \prod_{j=1}^{N}\left(1 - (\epsilon_j)^l\right)\right]}. \tag{2}$$

The stability condition of PRP is (1) with $\mu_N = \mu_{N,\text{PRP}}$.

IV. SIMPLE NETWORK CODING

We next consider SNC, with the following queue system *for each group*. The group leader keeps a queue Q_0 storing all newly arrived packets. If a packet transmitted from Q_0 is received by all N receivers, the packet leaves the system. If the packet is received by all receivers except receiver k, then the packet leaves Q_0 and enters Q_k at the group leader. When Q_0 is empty, we form a network-coded packet from all head-of-line packets in queues $\{Q_k\}_{k=1}^{N}$ by XOR-ing all of them and broadcast it to all receivers in the group. If some Q_k's are empty, we let the corresponding head-of-line packets to be all-zero dummy packets, where we assume ordinary packets cannot be all-zero.[2] If a packet is lost at at least two receivers, then we consider the packet to be lost at *all* receivers and it remains at Q_0.

Theorem 1: The service rate of SNC is

$$\mu_{N,\text{SNC}} = \frac{\left[1 + \sum_{l=1}^{N}\frac{\epsilon_l}{1-\epsilon_l}\right]\prod_{j=1}^{N}(1-\epsilon_j)}{1 - \epsilon_N + \epsilon_N\prod_{j=1}^{N-1}(1-\epsilon_j)}(1 - \epsilon_N) \tag{3}$$

for each group of N terminals.

Proof: Packet transmissions can be modeled as a Markov chain with two states. The initial state 1 corresponds to the case when the packet under transmission is not received by at least two receivers. If the packet is received by all but one receiver, then state 2 is reached. Otherwise, if the packet is received by all receivers, then the packet leaves the system and

we are back to state 1. Let $p_{i,j}$ be the transition probability from state i to state j, and t_i be the service time of a packet starting from state i. Then, the expected service time is given by

$$T_{N,\text{SNC}} = \mathbb{E}[t_1] = 1 + \sum_{i=1,2} p_{1,i} \mathbb{E}[t_i]. \tag{4}$$

In state 2, the probability that any network-coded packet is delivered to the receivers is upper bounded by the largest erasure probability ϵ_N. In general, additional time should be needed for the packets in Q_k for all $k \neq 0$. However, this is not necessary for SNC because the service time of the packets in Q_k for any $k \neq 0$ are all superimposed by means of network coding. For $k \neq 0$, the arrival rate of Q_k is

$$\frac{\lambda \epsilon_k \prod_{j=1,j\neq k}^{N}(1-\epsilon_j)}{\prod_{j=1}^{N}(1-\epsilon_j) + \sum_{l=1}^{N} \epsilon_l \prod_{j=1,j\neq l}^{N}(1-\epsilon_j)},$$

and the service rate of Q_k is $1-\epsilon_k$. Since $\{\epsilon_i\}$ is nondecreasing, Q_N has the largest arrival rate and the smallest service rate among all $\{Q_k\}_{k=1}^{N}$. At the verge of stability, Q_N will always be nonempty, any packet in $\{Q_k\}_{k=1}^{N-1}$ can be served with packets in Q_N simultaneously. As a result, it is sufficient to consider the case when the packets are delivered to receiver N. The time to deliver the network-coded packets to receiver N, namely t_2, is a geometric random variable with success probability $1-\epsilon_N$. Therefore, $\mathbb{E}[t_2] = \frac{1}{1-\epsilon_N}$.

The expected service time can be expressed from (4) as

$$T_{N,\text{SNC}} = 1 + \left[\epsilon_N \prod_{j=1}^{N-1}(1-\epsilon_j) \right] \frac{1}{1-\epsilon_N}$$

$$+ \left[1 - \prod_{j=1}^{N}(1-\epsilon_j) - \sum_{l=1}^{N} \epsilon_l \prod_{j=1,j\neq l}^{N}(1-\epsilon_j) \right] T_{N,\text{SNC}}. \tag{5}$$

The service rate is then given by $\mu_{N,\text{SNC}} = \frac{1}{T_{N,\text{SNC}}}$. ∎

The stability condition of SNC is (1) with $\mu_N = \mu_{N,\text{SNC}}$.

SNC improves the stable rate over PRP but it is still inefficient, as we will discuss next. In the following we present two methods to improve on its performance.

V. OPPORTUNISTIC NETWORK CODING

In SNC, suppose the group leader transmits network-coded packets. Since Q_k contains packets that are not received by receiver k only, if $\{Q_k\}_{k=1}^{i}$ are all empty for some $i < N$, then the network-coded packets should be formed from packets in $\{Q_k\}_{k=i+1}^{N}$ and intended for receivers $i+1, i+2, \ldots, N$ only. Receiver i has the packets in $\{Q_k\}_{k=i+1}^{N}$, so it can construct and transmit the network-coded packets. Hence, network coding should be done *opportunistically*. This is consistent with using the side information (obtained through opportunistic listening) to improve the network coding performance [10].

In ONC, we consider the following queue system *in each group*. At the group leader, there are queues $\{Q_k\}_{k=0}^{N}$ as in SNC. We introduce additional queues $Q_{i,k}$ with $1 \leq i < k \leq N$. The queue $Q_{i,k}$ resides at receiver i and stores packets

that are not received by receiver k only. Hence, $Q_{i,k}$ stores the same packets as Q_k does.

ONC operates as follows. When Q_0 is nonempty, packets are transmitted from Q_0. If it is empty, network coding is used. If Q_1 is nonempty, network-coded packets are formed and transmitted from the group leader as in SNC. If there exists an i such that $\{Q_k\}_{k=1}^{i}$ are all empty and Q_{i+1} is nonempty, network-coded packets are formed by XOR-ing the head-of-line packets in $\{Q_{i,k}\}_{k=i+1}^{N}$ and transmitted from receiver i.

Theorem 2: The service rate of ONC is

$$\mu_{N,\text{ONC}} = \frac{\left[1 + \sum_{l=1}^{N} \frac{\epsilon_l}{1-\epsilon_l} \right] \prod_{j=1}^{N}(1-\epsilon_j)}{1 + \sum_{l=1}^{N} \rho_{l,N}} \tag{6}$$

for each group of N terminals, where

$$\rho_{k,N} = \eta_{k,N} \frac{1-\epsilon_k}{1-\epsilon_1} - \sum_{l=1}^{k-1} \rho_{l,N} \frac{1-\epsilon_{k-l+1}}{1-\epsilon_1}, \tag{7}$$

the summation when $k=1$ is the empty sum and

$$\eta_{k,N} = \epsilon_k \prod_{j=1,j\neq k}^{N}(1-\epsilon_j) \frac{1}{1-\epsilon_k}. \tag{8}$$

Proof: The quantity $\eta_{k,N}$ is the expected service time to clear all packets in Q_k *if they are transmitted by the group leader*. From this, we calculate $\rho_{k,N}$, the expected service time due to network coding when Q_k is nonempty and $\{Q_j\}_{j=1}^{k-1}$ are all empty. The expected service time of ONC is given by

$$T_{N,\text{ONC}} = 1 + \sum_{l=1}^{N} \rho_{l,N}$$

$$+ \left[1 - \prod_{j=1}^{N}(1-\epsilon_j) - \sum_{l=1}^{N} \epsilon_l \prod_{j=1,j\neq l}^{N}(1-\epsilon_j) \right] T_{N,\text{ONC}}. \tag{9}$$

This is the same as (5), except the second term on RHS of (9) is replaced by the expected service time due to network coding when it is done opportunistically. If Q_1 is nonempty, network coding is performed at the group leader, so

$$\rho_{1,N} = \eta_{1,N}.$$

To compute $\rho_{2,N}$, we first compute the difference $\eta_{2,N} - \rho_{1,N}$, the expected time to transmit the packets in Q_2 by the group leader that are not network-coded with packets in Q_1 because Q_1 is empty. These packets are transmitted by receiver 1 at rate $1-\epsilon_1$ in ONC instead of $1-\epsilon_2$ by the group leader. So,

$$\rho_{2,N} = (\eta_{2,N} - \rho_{1,N}) \frac{1-\epsilon_2}{1-\epsilon_1}.$$

Similarly, we can compute

$$\rho_{3,N} = \left((\eta_{3,N} - \rho_{1,N}) \frac{1-\epsilon_3}{1-\epsilon_2} - \rho_{2,N} \right) \frac{1-\epsilon_2}{1-\epsilon_1}$$

$$= \eta_{3,N} \frac{1-\epsilon_3}{1-\epsilon_1} - \left(\rho_{1,N} \frac{1-\epsilon_3}{1-\epsilon_1} + \rho_{2,N} \frac{1-\epsilon_2}{1-\epsilon_1} \right),$$

and in general, we have (7). The service rate is given by $\mu_{N,\text{ONC}} = \frac{1}{T_{N,\text{ONC}}}$. ∎

The stability condition of ONC is (1) with $\mu_N = \mu_{N,\text{ONC}}$.

$$\mu_{N,\text{SNCOF}} = \frac{1 - \epsilon_1 + \epsilon_1 \prod_{j=2}^{N}(1-\epsilon_j)}{1 + \epsilon_N \prod_{j=1}^{N-1}(1-\epsilon_j)\frac{1}{1-\epsilon_N} + \sum_{l=2}^{N-1} \epsilon_l \prod_{j=1}^{l-1}(1-\epsilon_j)\left[1 - \prod_{j=l+1}^{N}(1-\epsilon_j)\right]\frac{1}{\mu_{N-l+1,\text{SNCOF}}}} \tag{10}$$

$$\mu_{N,\text{ONCOF}} = \frac{1 - \epsilon_1 + \epsilon_1 \prod_{j=2}^{N}(1-\epsilon_j)}{1 + \sum_{l=1}^{N} \rho_{l,N} + \sum_{l=2}^{N-1} \epsilon_l \prod_{j=1}^{l-1}(1-\epsilon_j)\left[1 - \prod_{j=l+1}^{N}(1-\epsilon_j)\right]\frac{1}{\mu_{N-l+1,\text{ONCOF}}}} \tag{12}$$

VI. Network Coding Combined with Opportunistic Forwarding

In SNC, if a packet is received by all receivers $j \leq i$, where $i \leq N - 2$, it is considered to be lost at all receivers. Since receiver i already has the packet, receiver i can forward it on behalf of the group leader. Therefore, we should consider combining SNC with opportunistic forwarding.

The queue structure for SNCOF for each group is as follows. We have queues $\{Q_k\}_{k=0}^{N}$ at the group leader as in SNC. For receiver i, $i \leq N - 2$, we include additional queues Q_0^i and $\{Q_k^i\}_{k=i+1}^{N}$. Consider a *reduced* group that consists of receivers $i+1, \ldots, N$ with receiver i acting as the group leader of this reduced group. Q_0^i stores the packets that are already received by receivers $1, \ldots, i$ but not receiver $i+1$, and are not handled by SNC. We assume packets in Q_0^i are not received by any receiver in the reduced group. Q_k^i stores packets that are already received by all receivers in the reduced group except receiver k. The roles of the queues Q_0^i and $\{Q_k^i\}_{k=i+1}^{N}$ are the same as the queues $\{Q_k\}_{k=0}^{N}$ in the group leader.

SNCOF works as follows. When Q_0 is nonempty, packets are transmitted from Q_0. If Q_0 is empty, we use SNC with queues $\{Q_k\}_{k=1}^{N}$. If all queues in the group leader and receivers $1, \ldots, i-1$ are empty, but not all queues in receiver i are empty, then we consider the reduced group with receiver i as the group leader. Packets are transmitted from Q_0^i if it is nonempty; otherwise, we use SNC with queues $\{Q_k^i\}_{k=i+1}^{N}$.

Theorem 3: The service rate of SNCOF is given by (10) for each group of N terminals.

Proof: The expected service time of SNCOF is

$$T_{N,\text{SNCOF}} = 1 + \left[\epsilon_N \prod_{j=1}^{N-1}(1-\epsilon_j)\right]\frac{1}{1-\epsilon_N}$$
$$+ \sum_{l=2}^{N-1}\left[\epsilon_l \prod_{j=1}^{l-1}(1-\epsilon_j)\left(1 - \prod_{j=l+1}^{N}(1-\epsilon_j)\right)T_{N-l+1,\text{SNCOF}}\right]$$
$$+ \left[1 - \prod_{j=1}^{N}(1-\epsilon_j) - \epsilon_1\prod_{j=2}^{N}(1-\epsilon_j) - \sum_{l=2}^{N}\epsilon_l\prod_{j=1}^{l-1}(1-\epsilon_j)\right]$$
$$\times T_{N,\text{SNCOF}}. \tag{11}$$

The third term on RHS of (11) is the expected service time due to opportunistic forwarding. For a packet that is received by all receivers $j < l$ and is lost at receiver l, if it is received by all remaining receivers, it will be delivered by SNC; otherwise it will be delivered by opportunistic forwarding in a reduced system that consists of only $N - l + 1$ receivers. The last term on the RHS of (11) is changed to include only packets that

are not received by all receivers and do not participate in SNC or opportunistic forwarding. Upon simplification of (11), the service rate $\mu_{N,\text{SNCOF}} = \frac{1}{T_{N,\text{SNCOF}}}$ is given by (10). ∎

The stability condition for SNCOF is (1) with $\mu_N = \mu_{N,\text{SNCOF}}$, where $\mu_{N,\text{SNCOF}}$ can be found recursively by (10).

We can also combine ONC with opportunistic forwarding, since the packets that are taken care of by opportunistic forwarding in SNCOF are also ignored by ONC. Hence, both network coding and forwarding should be done opportunistically. We can simply modify SNCOF by using ONC everywhere instead of SNC to form a new policy, ONCOF.

Theorem 4: The service rate of ONCOF is given by (12) for each group of N terminals, where $\rho_{l,N}$ is given by (7).

Proof: The expected service time of ONCOF is

$$T_{N,\text{ONCOF}} = 1 + \sum_{l=1}^{N} \rho_{l,N}$$
$$+ \sum_{l=2}^{N-1}\left[\epsilon_l \prod_{j=1}^{l-1}(1-\epsilon_j)\left(1 - \prod_{j=l+1}^{N}(1-\epsilon_j)\right)T_{N-l+1,\text{ONCOF}}\right]$$
$$+ \left[1 - \prod_{j=1}^{N}(1-\epsilon_j) - \epsilon_1\prod_{j=2}^{N}(1-\epsilon_j) - \sum_{l=2}^{N}\epsilon_l\prod_{j=1}^{l-1}(1-\epsilon_j)\right]$$
$$\times T_{N,\text{ONCOF}}, \tag{13}$$

where we replaced the second term on RHS of (11) by the expected service time due to network coding when it is done opportunistically to get (13). Upon simplification of (13), the service rate $\mu_{N,\text{ONCOF}} = \frac{1}{T_{N,\text{ONCOF}}}$ is given by (12). ∎

The stability condition for ONCOF is (1) with $\mu_N = \mu_{N,\text{ONCOF}}$, where $\mu_{N,\text{ONCOF}}$ can be found recursively by (12).

VII. Overhead and Feedback Requirement

We compute the overhead for each policy, *i.e.*, the number of bits needed to indicate which receivers are the intended ones and notify those receivers what packet is transmitted. The following only apply for $N > 1$, since all policies reduce to hop-by-hop forwarding when $N = 1$, thus no overhead is incurred.

- *PRP*: The group leader needs to inform the receivers whether the transmitted packet is a new transmission, or not. This can be handled by alternating one control bit depending on whether all receivers have already received the packet under transmission, or not.
- *SNC*: Any network-coded packet is formed from the head-of-line packets in queues $\{Q_k\}_{k=1}^{N}$. When a network-coded packet is transmitted, if a receiver decodes

the packet to get a dummy packet, it can simply discard the packet. Hence, any transmitted packet is intended for all receivers and we only have to distinguish whether it is a network-coded packet or not, *i.e.*, one bit is needed for the overhead.

- *ONC*: When receiver j receives a network-coded packet from receiver $i < j$, receiver j can decode the packet as if it were transmitted from the group leader, with $\{Q_k\}_{k=1}^i$ all empty. Hence, one bit of overhead is required as in SNC.
- *SNCOF*: SNCOF can be considered as SNC applied to $N-1$ groups, either the whole group or the reduced group with receiver i being the group leader for $i \leq N - 2$. Therefore, a total of $1 + \lceil \log_2(N-1) \rceil$ bits are required for the overhead.
- *ONCOF*: By similar reasoning as for ONC and SNCOF, the overhead required for ONCOF is the same as that of SNCOF, *i.e.*, $1 + \lceil \log_2(N - 1) \rceil$ bits.

Next we discuss the amount of feedback required. The feedback needs to inform every terminal in the group and the group leader which receivers receive the packet transmitted in the current time slot. Assuming perfect channel feedback, this can be done by letting each receiver broadcast one bit indicating whether it receives a packet in the current time slot or not. Therefore, the amount of feedback is equal to the number of terminals in a group, which is independent of the policies we use.

VIII. EFFECTS OF NETWORK CODING AND TOPOLOGY CONTROL ON STABLE THROUGHPUT

For illustration, suppose the probability of successful transmission varies with the number of hops according to some power law, *i.e.*, $1 - \epsilon_i \propto i^{-\alpha}$ with $\alpha > 0$, and we only allow transmissions with erasure probability at most $\epsilon_{\text{th}} > 0$. If we divide the network in groups of N terminals, this means $\epsilon_N \leq \epsilon_{\text{th}}$. We assume $\epsilon_N = \epsilon_{\text{th}}$ in the following. Then

$$\epsilon_i = \max\{1 - (1 - \epsilon_{\text{th}})(N/i)^\alpha, 0\}, i < N, \qquad (14)$$

where $\max\{\cdot, \cdot\}$ is included to satisfy $\epsilon_i \geq 0$. Figs. 2 and 3 show the maximum stable rates of PRP, SNC, ONC, SNCOF and ONCOF, with different values of α and ϵ_{th}. The packet length is taken to be sufficiently large so that the overhead is insignificant. The following observations are made:

- The maximum stable rate starts with $\frac{1}{3}(1 - \epsilon_{\text{th}})$ at $N = 1$, which corresponds to hop-by-hop forwarding.
- The performance of ONCOF is the best, followed by ONC or SNCOF, and SNC is the worst.
- For small N, ONC has better performance than SNCOF; but as N increases, SNCOF is better than ONC.
- The optimal cluster size can be greater than one, *e.g.*, four in Fig. 2 and two in Fig. 3.

From (14), $\epsilon_i = 0$ may occur if N is sufficiently greater than i. This means a high transmission power is used to reach the receiver N hops away, then the received power at the receiver
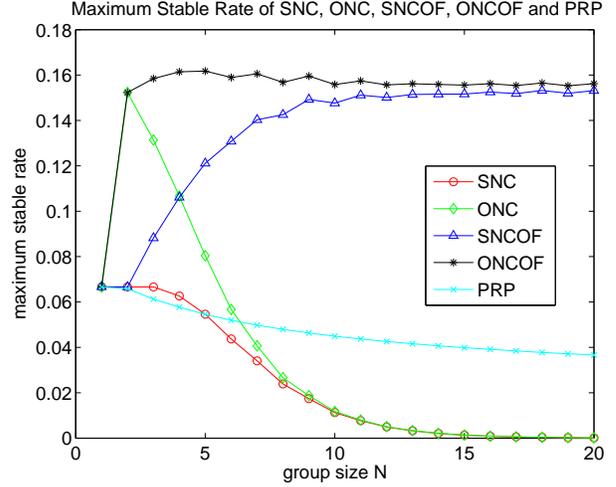
Fig. 2. Maximum stable rate against group size, with $\alpha = 2$ and $\epsilon_{\text{th}} = 0.8$.
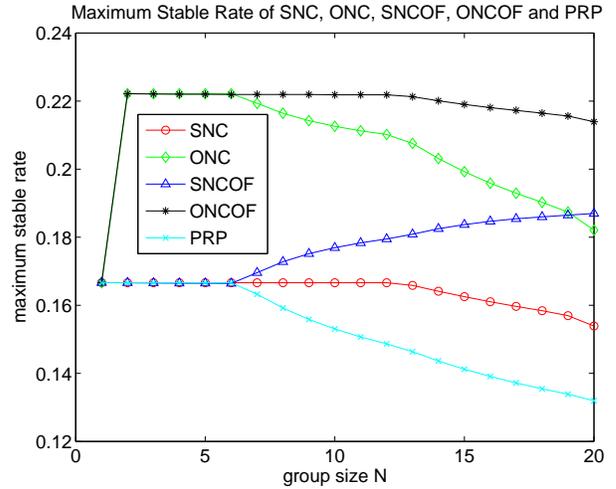
Fig. 3. Maximum stable rate against group size, with $\alpha = 4$ and $\epsilon_{\text{th}} = 0.5$.

i hops away is high enough that the channel becomes error-free. If $\epsilon_i = 0$ for all $i < N$, it is possible for the maximum rates to be leveled off as shown in Fig. 3 for small N.

Consider the inequality $\sum_{l=1}^{N-1} \epsilon_l \prod_{j=1, j \neq l}^{N-1} (1 - \epsilon_j) + \prod_{j=1}^{N-1} (1 - \epsilon_j) \leq 1$, meaning the probability that a packet is lost at at most one receiver when a transmitter broadcasts to $N-1$ receivers must be no more than one. This holds with equality *for any* $\{\epsilon_i\}$ if and only if $N = 1, 2$, where the empty sum is zero and the empty product is one. From this we obtain

$$\frac{\left[1 + \sum_{l=1}^N \frac{\epsilon_l}{1-\epsilon_l}\right] \prod_{j=1}^N (1 - \epsilon_j)}{1 - \epsilon_N + \epsilon_N \prod_{j=1}^{N-1}(1 - \epsilon_j)} \leq 1.$$

Together with (3), the maximum stable rate of SNC is the largest when $N = 1, 2$ *for any* $\{\epsilon_i\}$, implying the optimal N for SNC is one or two. The above argument also holds if ϵ_{N-1} and ϵ_N are the only possible nonzero elements in $\{\epsilon_i\}$. Hence the maximum rate of SNC levels off between $N = 1$
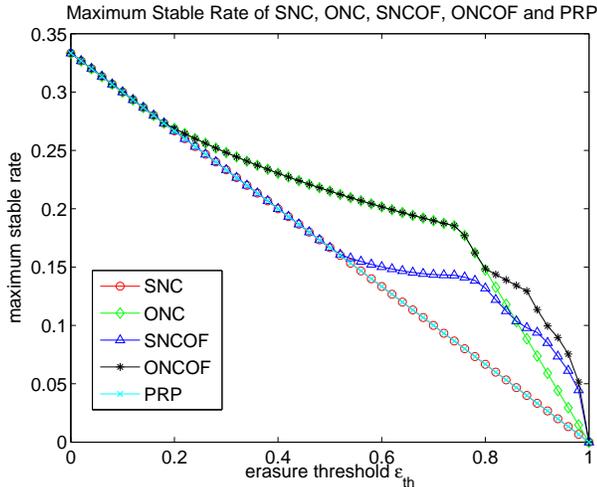
Fig. 4. Maximum stable rate against erasure threshold, with $\alpha = 2$.
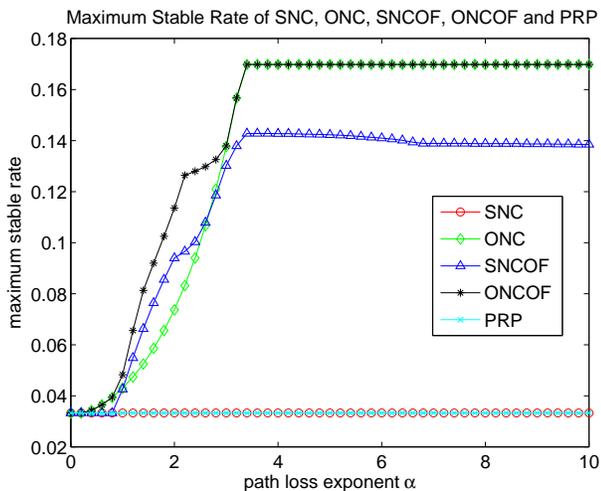
Fig. 5. Maximum stable rate against path loss exponent, with $\epsilon_{\text{th}} = 0.9$.

and $N = 12$ in Fig. 3. From (2), the maximum stable rate for PRP is the largest when $N = 1$ *for any* $\{\epsilon_i\}$. Therefore, the optimal N for PRP is one. The maximum stable rate can also be achieved when ϵ_N is the only nonzero element in $\{\epsilon_i\}$. For ONC, SNCOF and ONCOF, it is possible to have a larger maximum stable rate by increasing N as in Figs. 2 and 3.

Fig. 4 shows the maximum rate (optimized over N) against erasure probability threshold ϵ_{th} when $\alpha = 2$. The maximum stable rate is always achieved at $N = 1$ for PRP and SNC, *i.e.*, $\lambda_{\text{max,PRP}} = \lambda_{\text{max,SNC}} = \frac{1}{3}(1 - \epsilon_{\text{th}})$. For ONC, SNCOF and ONCOF, the improvement over PRP and SNC is minimal when ϵ_{th} is too small, *e.g.*, $\epsilon_{\text{th}} < 0.2$ in Fig. 4. As ϵ_{th} increases, ONC has a larger maximum stable rate than SNCOF, but eventually SNCOF outperforms ONC. Erasure probabilities ϵ_i, $i < N$, increase as $\epsilon_N = \epsilon_{\text{th}}$ increases. When ϵ_{th} is small, most packets will have a high probability to be lost at at most one

receiver, hence a higher rate can be achieved by ONC rather than SNCOF (in which these packets are actually delivered by SNC). As ϵ_{th} increases, packets that are received by all but one receivers will be less likely to appear. Since ONC only helps in delivering these packets, the benefits of ONC drop when ϵ_{th} is too large. ONCOF combines both ONC and SNCOF to achieve the best performance among all policies.

Fig. 5 shows the maximum rate (optimized over N) against path loss exponent α when $\epsilon_{\text{th}} = 0.9$. For PRP and SNC, $\lambda_{\text{max,PRP}} = \lambda_{\text{max,SNC}} = \frac{1}{3}(1 - \epsilon_{\text{th}})$, independent of α. SNCOF achieves a higher maximum stable rate than ONC for small α, and the opposite holds as α increases. Since $\epsilon_N = \epsilon_{\text{th}}$ is fixed, the erasure probabilities ϵ_i, $i < N$, decrease as α increases. Therefore, the observations when α increases are similar to those when ϵ_{th} decreases.

IX. CONCLUSION

We considered network coding for broadcasting in a linear wireless erasure network. We presented different network coding policies with opportunistic forwarding and topology control, and derived the maximum achievable rate while keeping the packet queues stable. For each transmission policy, we clustered terminals into groups of fixed size and applied network coding within each group. Then, the optimal cluster size was found to optimize the achievable rates by taking into account the packet overhead necessary for network coding. The results showed that the throughput gain improves with opportunistic network coding and benefits from topology control in terms of choosing transmission and interference ranges.

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