Spectrum Measurement Markets for Tiered Spectrum Access

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Abstract

The recent framework for tiered spectrum sharing in the 3.5 GHz band allows for Environment Sensing Capability operators (ESCs) to measure spectrum occupancy so as to enable commercial users to use this spectrum when federal incumbent users are not present. Motivated by this, we consider a scenario in which two spectrum access firms (SAs) seek to access a shared band of spectrum and must in turn purchase spectrum measurements from one of two ESCs. Given the purchased measurements, the SAs compete on price to serve customers. We consider both the case where both SAs seek to access the same shared band of spectrum and the case where they each have a portion of this band that they can exclusively use on a secondary basis. We study how differences in licensing approaches, the ESC's prices and the quality of the spectrum measurements impact the resulting market equilibrium between the SAs. In particular, we show that when the SAs share a single band of spectrum, having different qualities of measurements available to different SAs can lead to better economic welfare. When each has a separate licensed band, this difference does not matter.

I. Introduction

Recently, the FCC in the U.S. has finalized plans for the Citizens Broadband Radio Service (CBRS) [1]. These plans will enable commercial users to share the 3.5GHz band with federal incumbent users including naval radar and satellite services. Sharing this band in a given location is to be controlled by one or more Spectrum Access Systems (SASs), which are geographical databases that coordinate usage of the band. It is envisioned that in many areas multiple companies will operate approved SASs.[1] Companies wishing to offer service in that

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[1]In the first round of applications, six different companies requested that the FCC approve them as SAS operators [2].

band must then register with one SAS. Additionally, each SAS can utilize an environmental sensing capability operator (ESC). These ESCs will consist of a network of sensors used to detect the presence (or absence) of federal incumbent users. An ESC that can deliver high quality measurements will enable a SASs to allow its customers to access the spectrum band more frequently. For example, without such sensing, an SAS may be forced to adopt overly conservative exclusions zones to prevent interference to incumbents.

An interesting feature of the CBRS ecosystem is that there are multiple levels of competition that may emerge. Multiple ESCs may compete to sell their spectrum measurements to different SASs, who in turn may compete for registering different service providers in a given area.[2] These service providers in turn may be competing to offer wireless services to end users. Furthermore, different ESCs may offer different qualities of sensing, in which case the choice of ESC will in turn impact the quality of service offered by the downstream firms. There are many questions that arise in such a setting. In general, it is not clear if multiple ESCs would be able to co-exist in the market, and if so, what is the impact of the quality of their information on their market share? Likewise, would multiple SASs or service providers exist in the market? Does encouraging such competition improve economic welfare?

In this paper, we consider a stylized model motivated by the CBRS ecosystem to gain insights into the above questions. We consider a model with two tiers and two firms at each tier. At one tier are two ESCs, who offer spectrum measurement data to Spectrum Access firms (SAs).[3] Given this information, two SAs in turn compete to serve end-users in a given area whenever the ESC data tells them the spectrum is available. We focus on a single geographic area and assume both SAs only use a single shared band of spectrum (i.e., neither SA has access to other bands of spectrum). We focus on the case where the ESCs have different information regarding the presence of the incumbent because of different sensing capabilities. The SAs can obtain information from at most one of the ESCs. If a SA does not obtain information from any of the ESCs, we assume that it can not offer service to the customers (i.e., this could model a situation where the given location is within an exclusion zone). We then analyze a multi-stage game in which the SAs first decide on contracting with an ESC. Given these decisions, the SAs then

[2]For example as of February, 2018, four different entities have been conditionally approved as ESC operators [3].

[3]We can view an SA as either an SAS provider selling access directly to end users or a wireless service provider in a market in which the ESC and SAS provider are a single firm.

compete for users.

We consider two possibilities for how the SAs utilize the spectrum band when it is available. First, we consider the case where both SAs share this entire band of spectrum, which we refer to as *unlicensed (secondary) access*. We also consider the case where this band is divided into two equal sized sub-channels and each SA exclusively utilizes one of these sub-channels. This second approach we refer to as *licensed (secondary) access*. Our model of unlicensed access is motivated by the General Authorized Access (GAA) tier in the CBRS system, while licensed access is motivated by the Priority Access (PA) tier.

To model the competition among the SAs we adopt a similar framework as that used in [4]–[8] to study competition among wireless service providers using unlicensed and/or licensed spectrum. This in turn is based on models used to study price competition with congestible resources (e.g. [9], [10]). As in these models, we assume that SAs compete by offering prices for their service. There is a continuum of non-atomic users who in turn select a SA based on the *delivered price* given by the sum of the announced service price and a congestion cost which increases in the number of users using the band of spectrum in which the SA is offering service. Our work differs from this prior work in that the given band of spectrum is not always available and that this availability is in turn driven by the information a SA acquires from an ESC.

We begin in Section II by considering the case of unlicensed access and a user population that is homogeneous in how they value service. Interestingly, our analysis shows that the two SAs never chose to obtain information from the same ESC (Theorem 8). If both ESC's offer the same quality only one SA will purchase this information and the other will stay out of the market. However, if the ESCs offer different qualities of information, both SAs may enter the market, each obtaining information from one ESC. Moreover, in this case, more customers will be served. In Section IV, we show that these insights still hold when users have heterogeneous valuations. In Section V, we turn to the case where the spectrum is licensed. In this case, we find that it is never possible in equilibrium for the SAs to obtain information from different ESCs, though it is possible for them to both to acquire information from the same ESC (or for only one to acquire information). Hence, only one of the ESCs will offer information, and the other will stay out of the market. Thus, licensed spectrum does not lead to a competition among the ESCs. This illustrates that the spectrum sharing policy has a strong impact on the type of market structure that emerges.

We also show that the amount of shared bandwidth is another important parameter in determin-

TABLE I

FREQUENTLY USED NOTATIONS

q_j	Probability ESC j indicates spectrum available.
$\tilde{p}_j$	Price set by ESC j.
v	Homogeneous user valuation.
W	Total bandwidth.
π_i	Payoff of SA i.
Λ	Total number of users.
λ_i	Number of SA i's subscribers.
p_i	Price set by SA i.

ing the resulting market structure. We model increasing bandwidth as a decrease in the congestion cost for a given number of users (similar to [6], [7]). We show that both with unlicensed access (Corollary 1) and licensed access (Corollary 2), the impact of increasing bandwidth is similar. Interestingly, if the amount of bandwidth is too large, only one of the SAs will exist in the market (i.e., there will be no competition). On the other hand, if the amount of bandwidth is too small, no SAs can offer service.

In addition to the aforementioned work on price competition among wireless service providers, other related work includes a growing literature that studies the role of information acquisition on competition. For example, [11] considers acquiring information about a competitor's supply in a spectrum sharing scenario, and [12] studies firms that can acquire information about customer demand from a third party. The question of whether firms should share information with competitors has also received attention (e.g [13]). Another line of related work is that addressing issues around the design of the ESC and SAS infrastructure. This includes work on sensor deployment (e.g. [14]) and work on privacy issues raised by such networks (e.g. [15], [16]).

II. System Model for Unlicensed Access and Homogeneous Users

We begin in this section by stating our model for the case of unlicensed access and homogeneous users. We will relax each of these assumption in subsequent sections. The notation used in this model is also summarized in Table I.

We consider a model in which there are two ESCs (denoted by ESC A and ESC B) and two SAs denoted by SA 1 and SA 2. Each SA seeks to serve users in a given band of spectrum at a given location. To do this, the SAs must acquire spectrum measurements from one of the ESCs and can only use the spectrum when the ESC indicates that it is available (i.e., not being used by a federal incumbent). If an SA does not acquire information from either ESC, we assume it can not serve any users. If both SAs receive information that the spectrum is available, then they both can utilize it. We next discuss the participants in this market in more detail.

A. Information Selling from the ESC

Each ESC provides a binary indication of whether the spectrum is available for use over time based on their own sensing capabilities. We assume that each ESC must be certified to have a negligible probability of missed detection of the incumbent, i.e., if the incumbent is present, the ESC will never announce that the spectrum is available. However, we do allow the ESCs to incur false alarms, i.e., if the incumbent is not present, an ESC may still announce that the spectrum is not available. An ESC with better sensing capabilities will be less likely to make sure errors. We identify each ESC A and B with a probability q_A and q_B, respectively, that gives the probability that the ESC indicates that the spectrum is available (which in turn depends on the incumbent's usage patterns and the ESC's sensing capability). Without loss of generality, we assume that $q_A \geq q_B$ so that ESC A has the higher quality of information (unless they are identical). Further to simplify our exposition, we assume that ESC B's announcement is a degraded version of ESC A, so that whenever the ESC B indicates the channel is available, ESC A also does the same. However, when ESC A estimates the channel is available, ESC B may not estimate the same.[4]

Each ESC will incur a cost providing its service due for example to the cost of building and operating its sensors and communicating information to a SA. We denote the cost incurred by ESC A and B by c_A and c_B, respectively. Since ESC A provides information with a higher accuracy, we assume that $c_A \geq c_B$ with equality only if $q_A = q_B$.

We assume that ESC A (B) sells its prediction to any of the SAs at the price $\tilde{p}_A$ ($\tilde{p}_B$). Here, $q_A, q_B, \tilde{p}_A$, and $\tilde{p}_B$ are common knowledge to both the ESCs and to the SAs. Throughout this paper, we also consider that $\tilde{p}_A$ and $\tilde{p}_B$ are exogenous parameters and focus on the strategic decisions of the SAs given these prices.

[4]Our analysis can easily be extended to the case where instead ESCs A and B make independent errors.

B. SAs Decisions

Each SA must make two decisions. First, it must decide whether to acquire information from ESC A, ESC B, or to not acquire any information at all. Second, if SA i acquires information, then it must decide on a price p_i that it will charge users for its service. We assume that these decisions are made in stages. In the first stage, both the SAs simultaneously decide on acquiring information. In the second stage, given the first stage decisions, the SAs then simultaneously choose prices to compete for users. Each SA i, seeks to maximize its profit given by

$$\pi_i = p_i \lambda_i - \tilde{p}_k \tag{1}$$

where λ_i indicates the number of users SA i serves and $\tilde{p}_k$ is the price it pays to acquire information from ESC k. If SA i decides not to acquire information in the first stage, then we set $\tilde{p}_k = 0$ and $\lambda_i = 0$ so that the overall profit is also zero, i.e., this models a case where SA i decides not to enter the market. This may occur when the revenue the SA would generate is not sufficient to recover the cost of acquiring information from one of the ESCs.

C. User's Subscription Model

We consider a mass Λ of non-atomic users, so that we have $\lambda_1+\lambda_2 \leq \Lambda$. Under our assumption of homogeneous users, each user obtains a value v for getting service from either SA. However, users also incur a cost for using the service, which as in [5]–[8] is given by the sum of the price charged to them by the SA and a *congestion cost* they incur when using this service. The congestion cost models the degradation in service due to congestion of network resources. We model the congestion cost for using a band of spectrum with bandwidth W by $g(x/W)$, where x is the total mass of users using that band and g is a convex, increasing function. Hence, the pay-off of a user receiving service from SA i is given by

$$v - p_i - g(x/W). \tag{2}$$

The dependence of g on W models the fact that a larger band of spectrum is able to support more users. The mass of users, x, using the band depends in turn on the licensing policy and the information available to the SAs. In this section we focus on unlicensed access, in which case users of both SAs utilize the same band of spectrum whenever both of them know that the spectrum is available. We model this as in [5]–[7], by setting $x = \lambda_1 + \lambda_2$. If there were more than 2 SAs using this band at a given, then this can be extended naturally by setting $x = \sum_{1 \in A} \lambda_i$, where A indicates the set of SAs using this band.

The SAs knowledge of spectrum availability in turn depends on the information they acquire from the ESCs. In particular, if SA i obtains information from ESC k and has λ_i users, these users are only able to use the spectrum when the ESC k reports the spectrum is available (which occurs with probability p_k). When users can not use the spectrum, we assume their pay-off is zero. When users can use the spectrum, they receive a pay-off as in (2), where the traffic of the other SA will in turn depend on the information that SA receives from its ESC. Hence, the pay-off obtained will be a random variable. We assume that users seek to maximize the expected value of this quantity.[5] Furthermore, users can choose not to purchase service from either SA, giving them a pay-off of zero.

The specific form of the average congestion will depend on which ESCs the SAs contract with. If both SAs obtain information from ESC A, then both SAs' customers will use the spectrum during the times ESC A specifies it is available (which occurs with probability q_A). In this case, the expected pay-off of any subscriber of SA i ($i = 1, 2$) is

$$q_A v - q_A g((\lambda_1 + \lambda_2)/W) - p_i. \tag{3}$$

Similarly, if both the SAs obtain information from the ESC B, the expected pay-off of any subscriber of SA i is $q_B v - q_B g(\lambda_1 + \lambda_2/W) - p_i$.

Next suppose that the SAs obtain information from different ESCs. Without loss of generality, assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESC B. Recall that when ESC B estimates that the channel is available, then ESC A also estimates that the channel is available. Thus, the subscribers of SA 2 always face congestion from SA 1's users. However, the subscribers of SA 1 only face congestion from SA 2's customers when ESC B also indicates that the incumbent is not present (which occurs with probability q_B). Thus, the subscribers of SA 1 enjoy an exclusive access to the spectrum with probability $q_A - q_B$. This results in these users having an expected pay-off of

$$q_A v - (q_A - q_B) g(\lambda_1/W) - q_B g(\frac{\lambda_1 + \lambda_2}{W}) - p_1. \tag{4}$$

[5]For example, this is reasonable when users are purchasing service contracts with a long enough duration so that they see many realizations of the ESC reports.

On the other hand, a user of SA 2 will obtain an expected pay-off of

$$q_B v - q_B g(\frac{\lambda_1 + \lambda_2}{W}) - p_2. \tag{5}$$

Finally, suppose one SA i obtains information from ESC k, while the other SA chooses not to acquire information from either ESC (and so does not serve any customers). The users of SA i then obtain an expected pay-off of

$$q_k v - q_k g(\lambda_i / W) - p_i. \tag{6}$$

The expressions for the expected pay-offs can also be naturally extended to the case where there are multiple ESCs and SAs. For example, suppose that there were K ESCs labeled $A_1, A_2, \ldots, A_K$, with corresponding probabilities $q_1 > q_2 > \cdots > q_k$ and multiple SAs. Let $\mathcal{A}_k$ denote the set of SAs that acquire information from ESC A_1, A_2 up to A_k. The expected pay-off of users of an SA i obtaining information from ESC A_1 would be given by

$$q_1 v - \sum_{k=1}^{K} (q_k - q_{k+1}) g \left(\sum_{i \in \mathcal{A}_k} \lambda_i / W \right) - p_i,$$

where for convenience we define $q_{K+1} = 0$. Note as the number of ESCs and SAs grows, the number of ways that ESCs and SAs can be matched will grows exponentially, making a detailed analysis overly cumbersome.

D. Multi-Stage Market Equilibrium

We model the overall setting as a game with the SAs and the users as the players. Each SA's pay-off in this game is its profit (cf. (1)), while each user's objective is the expected pay-off described in Section II-C. This game consists of the following stages:

1) In the first stage, each SA selects one of the ESCs and pays $\tilde{p}_j$ $j = A, B$ or selects to stay out of the market.
2) In the second stage, SA i selects its price p_i knowing the decisions made in stage 1.
3) In the last stage, given the first two stages' decisions, the subscribers will choose one of the SAs from which to receive serve or choose not to receive service.

We refer to a sub-game perfect Nash equilibrium of this game as a *market equilibrium*.

III. Equilibrium Analysis for Unlicensed Access and Homogeneous Users

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In this section, we analyze the market equilibrium for our model with homogeneous users and unlicensed access via backward induction. We start with the final stage next.

A. User Equilibrium

In the final stage of the game, the *user equilibrium* specifies the subscribers λ_i of each SA i given the prices selected in the second stage and the ESC choices made in the first stage.

Each user is seeking to maximize its expected pay-off. Given our assumption of identical non-atomic users, the user equilibrium can be characterized as a *Wardrop equilibrium* [17]. More precisely, if in equilibrium both SAs are serving customers, then the expected pay-offs for both SA's must be the same (since, otherwise some customers would switch to the other SA). If one SA is not serving any customers, then its expected pay-off must be larger than that of the other SA. Additionally, this expected pay-off must be non-negative as otherwise some customers would be better off not purchasing service. Finally, if fewer than Λ customers are receiving service, then it must be that the expected pay-off is equal to zero as otherwise, some customers not receiving service would choose to receive service. We will refer to these properties as the *Wardrop equilibrium conditions*. It can be shown that these are necessary and sufficient for λ_1 and λ_2 to be a user equilibrium.

Suppose that both the SAs obtain information from the same ESC. In this case, customers of both SAs experience the same expected congestion cost, and so the expected customer pay-offs of the two SAs only differ in the announced prices. Using this and the Wardrop equilibrium conditions, we have the following characterization of the user equilibrium.[6]

Theorem 1. *Assume that both SAs obtain information from ESC j ($j \in \{1, 2\}$)*

1) *If $p_1 = p_2$ and $q_j v - q_j g(\Lambda/W) - p_1 \geq 0$, then any choice of λ_1 and λ_2 such that $\lambda_1 + \lambda_2 = \Lambda$ is a user equilibrium;*
2) *If $p_1 = p_2$ and $q_j v - q_j g(2\alpha/W) - p_1 = 0$ for some $\alpha < \Lambda/2$, then any choice of λ_1 and λ_2 such that $\lambda_1 + \lambda_2 = 2\alpha$ is a user equilibrium;*
3) *If $p_i > p_k$ (for $i \neq k$) and $q_j v - q_j g(\Lambda/W) - p_k \geq 0$, then the unique user equilibrium is $\lambda_k = \Lambda$ and $\lambda_i = 0$.*
4) *If $p_i > p_k$ (for $i \neq k$) and $q_j v - q_j g(\alpha/W) - p_k = 0$ for some $\alpha < \Lambda$, then the unique user equilibrium is $\lambda_k = \alpha$ and $\lambda_i = 0$.*

[6]Subsequently, we will show that the situation where both SA obtain information from the same ESC is not sustainable on the equilibrium path.

If $p_1 = p_2$, then as noted in this theorem the user equilibrium is not unique. However, the total number of subscribers in an equilibrium is unique. On the other hand, if one of the SAs sets a higher price i.e., $p_i > p_j$, then that SA will not receive any customers and there will be a unique number served by the other SA. Hence, the SA that selects a higher price will not have any revenue and so will have a negative profit due to the payment it makes to the ESC.

If the total number of customers served is less than Λ, then fixing the prices, the market coverage (given by the parameter α) is given by solving the corresponding equations in the 2nd and 4th cases in Theorem 1. Note that the solution to these equations will be increasing in q_i and W. This means that both SAs obtaining information from ESC A rather than ESC B will increase market coverage as will having more bandwidth available.

Next consider the case where only one SA acquires information from a ESC, while the other stays out of the market. The user equilibrium in this case is the same as that in case 3 or 4 of Theorem 1, where we can view the SA that does not acquire information as being the SA with the higher price, so that it won't be able to serve any customers.

Finally, consider when SA 1 and SA 2 obtain information from different ESCs. Without loss of generality, we assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESC B in the first stage. We can again obtain λ_1 and λ_2 from the Wardrop equilibrium conditions as is summarized in the following.

Theorem 2. *Assume SA 1 obtains information from ESC A and SA 2 obtains information from ESC B, the* unique *user equilibrium (λ_1, λ_2) satisfies:*

1) $\lambda_1 = \lambda_2 = 0$, *if* $q_A v - (q_A - q_B)g(0) - q_B g(0) - p_1 < 0$, *and* $q_B v - q_B g(0) - p_2 < 0$.
2) $\lambda_1 = \Lambda$, *if* $q_A v - (q_A - q_B)g(\Lambda/W) - q_B g(\Lambda/W) - p_1 \geq 0$, *and* $q_A v - (q_A - q_B)g(\Lambda/W) - q_B g(\Lambda/W) - p_1 \geq q_B v - q_B g(\Lambda/W) - p_2$.
3) $\lambda_1 = \alpha$, *if* $q_A v - (q_A - q_B)g(\alpha/W) - q_B g(\alpha/W) - p_1 = 0$ *and* $q_A v - (q_A - q_B)g(\alpha/W) - q_B g(\alpha/W) - p_1 > q_B v - q_B g(\alpha/W) - p_2$.
4) $\lambda_2 = \Lambda$, *if* $q_B v - q_B g(\Lambda/W) - p_2 \geq 0$ *and* $q_A v - (q_A - q_B)g(0) - q_B g(\Lambda/W) - p_1 < q_B v - q_B g(\Lambda/W) - p_2$.
5) $\lambda_2 = \alpha$, *if* $q_B v - q_B g(\alpha/W) - p_1 = 0$ *and* $q_A v - (q_A - q_B)g(0) - q_B g(\alpha/W) - p_1 < q_B v - q_B g(\alpha/W) - p_2$.
6) $\lambda_1 > 0, \lambda_2 > 0$ *such that* $\lambda_1 + \lambda_2 = \Lambda$, *if* $q_A v - (q_A - q_B)g(\lambda_1/W) - q_B g(\Lambda/W) - p_1 = q_B v - q_B g(\Lambda/W) - p_2$; *and* $q_A v - (q_A - q_B)g(\lambda_1/W) - q_B g(\Lambda/W) - p_1 \geq 0$.

7) $\lambda_1 > 0, \lambda_2 > 0$*, such that* $\lambda_1 + \lambda_2 = \alpha < \Lambda$ *where* $q_A v - (q_A - q_B) g(\lambda_1/W) - q_B g(\alpha/W) - p_1 = q_B v - q_B g(\alpha/W) - p_2$ *and* $q_B v - q_B g(\alpha/W) - p_2 = 0$.

In the first case in Theorem 2, users do not subscribe to any of the SAs . This is because even with no customers, the expected payoff to a user from both the SAs is negative.

Note that if $q_A v - (q_A - q_B) g(0) - q_B g(\lambda_2/W) - p_1 < 0$, then $\lambda_1 = 0$. Similarly, if $q_B v - q_B g(\lambda_2/W) - p_2 < 0$, then, $\lambda_2 = 0$. If $q_A v - (q_A - q_B) g(\Lambda/W) - q_B g(\Lambda) - p_1 > q_B v - p_2$, then $\lambda_1 = \Lambda$, and $\lambda_2 = 0$. Thus, in the cases 2 and 3 in Theorem 2, the subscribers only subscribe to SA 1 as the expected payoff attained by the users is positive for SA 1, but it is negative for SA 2 even when $\lambda_2 = 0$. On the other hand, in cases 4 and 5, the subscribers only subscribe to the SA 2 as the expected payoff attained by the users is positive for SA 2, but it is negative for SA 1 even when $\lambda_1 = 0$.

The number of subscribers is split between the two SAs when the expected payoff is the same in the Wardrop equilibrium. However, the market may or may not cover all the subscribers depending on the prices p_1, p_2, the probabilities q_A, q_B, and the valuation v. Also, the split of the market between the SAs is not arbitrary as in Theorem 1 - for a given set of prices there will now be a unique split satisfying the Wardrop equilibrium conditions. In this unique split, as the quality of information from ESC A increases (i.e., q_A increases), the market share of the SA 2 will decrease. On the other hand, if p_i increases, the market share of SA i will decrease.

B. Price Equilibrium

Next we turn to the second stage in which given the ESC choices, each SA i decides on its service price p_i with the goal of maximizing its profit as in (1). Note that in this stage any cost paid to an ESC in stage 1 is sunk, and so equivalently, in this stage the SAs seek to maximize their revenue given by $p_i \lambda_i$. When doing this, λ_i will be specified by the corresponding user equilibrium determined in the previous section, which in turn depends on if the SAs obtain information from the same ESC or a different ESC, or if one SA does not obtain information. We treat each of these cases separately.

First, we describe the case when both SAs obtain information from the same ESC.

Theorem 3. *If both the SAs obtain information from the same ESC, then in equilibrium* $p_1 = p_2 = 0$.

Essentially, in this case, both of the SAs are offering identical service using the same spectrum, which results in a “price war” leading the SAs to each try to undercut the other. At the resulting equilibrium, both the SAs set the price at 0. Note that in stage 1, both SAs will have incurred a cost of $\tilde{p}_j > 0$ to acquire information from the same ESC j. Hence, they will both have negative profits in such an equilibrium.[7] Later, we will show that such an outcome can not be sustained in an equilibrium path. We also note that this result easily generalizes to more SAs and ESCs. Namely, in a market with multiple ESCs and SAs, if two or more SAs acquire information from the same ESC and serve customers, then their prices will get competed to zero.

Next, we turn to the case where only one SA obtains information from an ESC. Thus, the SA will essentially be a monopolist when making its pricing decision. To facilitate our analysis of this case, we make the following assumption regarding the congestion costs:

Assumption 1. *Assume that $g(\cdot)$ is linear, i.e., $g(x/W) = x/W$.*

Throughout the rest of this paper, this Assumption 1 will be enforced.

The monopolistic price and profit are given in the following:

Theorem 4. *If SA i obtains information from the ESC j, while SA $k \neq i$ does not obtain information from either ESC, the unique equilibrium price for SA i is*

$$p_i^* = \max\{q_j v - q_j \Lambda/W, \frac{q_j v}{2}\}. \tag{7}$$

The third-stage user equilibrium is

$$\lambda_i^* = W \min\{v - \frac{p_i^*}{q_j}, \Lambda/W\}. \tag{8}$$

The monopolistic profit of the SAS i is

$$\pi_i = p_i^* \lambda_i^* - \tilde{p}_j. \tag{9}$$

The monopolistic profit in (9) can also be written as

$$\pi_i = \begin{cases} W q_j v^2/4 - \tilde{p}_j, & \text{if } v/2 \leq \Lambda/W, \\ q_j (v - \Lambda/W)\Lambda - \tilde{p}_j, & \text{otherwise.} \end{cases}$$

[7] If instead we assumed that each SA had to pay the ESC a marginal price for each customer (instead of a single flat price), then in equilibrium the prices will be set at the marginal price.

Note that though the first term in the expression of π_i is higher for $j = A$ as $q_A > q_B$, this does not necessarily mean that SA i will get a higher profit if it attains information from ESC A. This is because the price paid by the SA to obtain information from ESC A may be higher than that paid by the other SA, i.e., $\tilde{p}_A > \tilde{p}_B$. Clearly, if $\tilde{p}_A \leq \tilde{p}_B$, the profit attained by the SA will be higher if it selects ESC A. From (7), the price selected by SA i will be higher if it obtains information from ESC A. However, from (8), the market share (λ_i) is independent of the ESC selected by the SA. Thus, surprisingly, in a monopolistic scenario the number of users which receive service is independent of the choice of ESC made by the SA.

Next consider how the bandwidth W impacts the results in Theorem 4. As W increases, the profit of the SA increases. However, the rate of increase decreases when $v > 2\Lambda/W$. This is because when $v > 2\Lambda/W$, the monopoly SA serves whole market. Thus, the demand can not increase beyond that point, though a larger W can still enable the SA to increase its price. If $v < 2\Lambda/W$, the SA does not serve the whole market due to the high congestion cost.

Finally we consider the case there the SAs obtain information from different ESCs. Without loss of generality, we again assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESC B. Earlier, we showed that the equilibrium where both the SAs obtain information from the same ESC renders a negative profit to each of the SAs. Now, we show that under some conditions there exists a price equilibrium where both the SAs can get positive profits if they obtain information from different ESCs. Later, we will show that such a price equilibrium is sustainable along an equilibrium path.

Theorem 5. *Assume that SA* 1 *obtains information from ESC* A*, SA* 2 *obtains information from ESC* B*, and*

$$\frac{2\Lambda}{W} \geq v \geq \frac{2q_A + q_B}{W(q_A + 2q_B)}\Lambda \tag{10}$$

In this case there is a unique price equilibrium (p_1^*, p_2^*) *given by*

$$p_1^* = (q_A - q_B)\frac{(v + \Lambda/W)}{3}$$
$$p_2^* = (q_A - q_B)\frac{(2\Lambda/W - v)}{3}. \tag{11}$$

The user equilibrium (third-stage) is given by

$$\lambda_1 = W(\frac{v}{3} + \frac{\Lambda}{3W}), \quad \lambda_2 = W(\frac{2\Lambda}{3W} - \frac{v}{3}). \tag{12}$$

The profits of the SAs' are

$$\pi_1 = W(q_A - q_B)\left(\frac{\Lambda}{3W} + \frac{v}{3}\right)^2 - \tilde{p}_A.$$

$$\pi_2 = W(q_A - q_B)\left(\frac{2\Lambda}{3W} - \frac{v}{3}\right)^2 - \tilde{p}_B. \tag{13}$$

Note that the lower bound on v in (10) implies that $v > \Lambda$. Hence, the market share of SA 1 is higher than that of SA 2. The first term in the profit of SA 1 is also strictly larger compared to SA 2. However, SA 1 may have to pay more as $\tilde{p}_A$ maybe higher than $\tilde{p}_B$. Thus, SA 1's profit may be lower compared to that of SA 2. Also note that as the difference between q_A and q_B decreases, the profits of the SAs decrease. Intuitively, as the difference in the quality of the ESCs' information decreases, the SAs become competitive. When the qualities are equal, this becomes the same as if both SAs acquire information from a single ESC, which leads to a negative profit, as we have already seen in Theorem 3.

The sum of λ_1 and λ_2 in the equilibrium is equal to the total number of subscribers. Thus, wireless service is provided to every user. Hence, when the SAs obtain information from different ESCs and the condition in (10) is satisfied, the SAs select prices such that they cover the entire subscription base.

Also note that in contrast to the monopoly scenario, in this case, the consumer surplus is positive. Note from Theorem 5 that the lower bound on v in (10) is decreasing in W. Hence, as W decreases, the lower bound is unlikely to be satisfied. Thus, the situation is unlikely to arise when W is small. However, if W increases, the lower bound is more likely to be satisfied but the upper bound becomes tighter. Also, note that the price of both SAs decreases with W. For SA 1, this is offset by an increase in the number of users served, leading to SA 1's profit increasing with W. However, the number of users served by SA 2 decreases with W and, hence, so does its profit. For large enough W, the upper bound on v in (10) will become tight at which point SP 2 makes zero profit. The next corollary shows that when this bound is exceeded, at most one SA will enter the market.

Corollary 1. *If $v > \frac{2\Lambda}{W}$, there is no equilibrium where both the SAs will choose information from both the ESCs.*

Next we consider the price equilibrium when the lower bound on v in (10) is not satisfied.

Theorem 6. *Assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESA B. If*

$$v < \frac{2q_A + q_B}{W(q_A + 2q_B)}\Lambda$$

and $v > \dfrac{3q_A\Lambda}{W(4q_A - q_B)}$, then the unique price equilibrium (p_1^*, p_2^*) is given by

$$\begin{aligned} p_1^* &= q_A v/2 - q_B\frac{\Lambda}{2W} \\ p_2^* &= q_B(v - \Lambda/W). \end{aligned} \tag{14}$$

The third stage user equilibrium is

$$\begin{aligned} \lambda_1 &= \frac{q_A vW - q_B\Lambda}{2(q_A - q_B)}. \\ \lambda_2 &= \frac{(2q_A - q_B)\Lambda - q_A vW}{2(q_A - q_B)}. \end{aligned} \tag{15}$$

The profits of the SA's are

$$\begin{aligned} \pi_1 &= W(q_A v/2 - q_B\frac{\Lambda}{2W})^2\frac{1}{q_A - q_B} - \tilde{p}_A. \\ \pi_2 &= Wq_B(v - \Lambda/W)\frac{(2q_A - q_B)\Lambda/W - q_A v}{2(q_A - q_B)} - \tilde{p}_B. \end{aligned} \tag{16}$$

Note that when $q_A = q_B$, this case never arises.

Similar to Theorem 5, the total market share of the SAs cover the whole subscription base Λ. The price set by the SA 1 is higher compared to SA 2. However, the consumer surplus is zero unlike in Theorem 5. The market share of SA 1 is higher compared to the SA 2, however, in this case it is not double that of SA 2. The payoffs of the SAs are also lower compared to Theorem 5. This is because the condition stated in Theorem 6 is valid for a smaller range of v compared to Theorem 5.

If W is large or small, the condition stated in Theorem 6 can not be satisfied. In contrast to Theorem 5 as W increases, the prices increase. The subscription base of SA 1 and its profit increases as W increases, however, the subscription base and profit of SA 2 decreases.

Finally, we characterize the unique price equilibrium when $v \leq \dfrac{3q_A\Lambda}{4q_A - q_B}$.

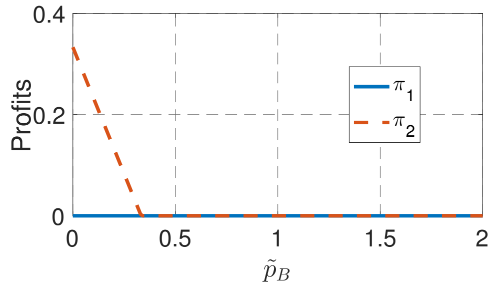

Fig. 1. The profits of each SA versus $\tilde{p}_B$ with $v = 10, \Lambda = 5, q_A = 0.5, q_B = 0.25, \tilde{p}_A = 4$.

Theorem 7. *Assume that SA 1 obtains information from ESC A and SA 2 obtains information from ESA B. If $v \leq \dfrac{3q_A\Lambda}{W(4q_A - q_B)}$, then a price equilibrium (p_1^*, p_2^*) is given by*

$$p_1^* = \frac{(q_A - q_B)v2q_A}{4q_A - q_B}$$
$$p_2^* = \frac{(q_A - q_B)vq_B}{4q_A - q_B}. \tag{17}$$

The third stage user equilibrium is given by

$$\lambda_1 = W\frac{v2q_A}{4q_A - q_B}, \lambda_2 = W\frac{vq_A}{4q_A - q_B}. \tag{18}$$

The SAs' profits are

$$\pi_1 = W(q_A - q_B)\left(\frac{v2q_A}{4q_A - q_B}\right)^2 - \tilde{p}_A,$$
$$\pi_2 = Wq_Bq_A(q_A - q_B)\left(\frac{v}{4q_A - q_B}\right)^2 - \tilde{p}_B, \tag{19}$$

In this equilibrium, the subscribers are again split among the two SAs. However, in contrast to Theorems 5 and 6, in Theorem 7 the entire market is not served. Similar to Theorem 5, in Theorem 7 the price set by SA 1 is higher compared to SA 2 and SA 1's market share is higher compared to SA 2. The profits of the SAs are lower compared to that obtained in Theorems 5 and 6 since the above result holds for smaller value of v.

The condition stated in Theorem 7 is satisfied when W is small. The condition for Theorem 5 is satisfied only when W is large. Hence, the subscription base of the SAs will cover the whole market only when W is large. In Theorem 7, the prices of the SAs are independent of W; however, the SA profits increase as W increases.

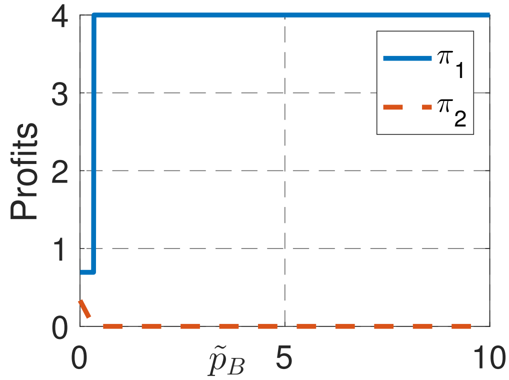

Fig. 2. The payoffs of each SA versus $\tilde{p}_B$ with $v = 8, \Lambda = 5, q_A = 0.5, q_B = 0.25, \tilde{p}_A = 4$.

C. ESC Selection Equilibrium

Now, we discuss the first stage equilibrium. Specifically, we state an equilibrium strategy which prescribes which ESC should be chosen by each SA.

Theorem 8. *In the first stage only one of the following four equilibria are possible:*

1) *Only one of the SAs obtains information from ESC A if* $(q_A - q_B)W\left(\frac{2\Lambda}{3W} - \frac{v}{3}\right)^2 < \tilde{p}_B$ *and* $W\max\{\frac{q_A v^2}{4}, q_A(v - \Lambda/W)\Lambda/W\} \geq \tilde{p}_A$.
2) *Only one of the SAs obtains information from ESC B if* $W\max\{\frac{q_B v^2}{4}, q_B(v - \Lambda/W)\Lambda/W\} \geq \tilde{p}_B$ *and* $(q_A - q_B)\left(\frac{\Lambda + v}{3}\right)^2 < \tilde{p}_A$.
3) *Both the SAs obtain information from different ESCs if* $W(q_A - q_B)\left(\frac{2\Lambda}{3W} - \frac{v}{3}\right)^2 \geq \tilde{p}_B$ *and* $W(q_A - q_B)\left(\frac{\Lambda}{3W} + \frac{v}{3}\right)^2 \geq \tilde{p}_A$.
4) *Neither SA chooses to obtain information from the ESCs if* $W\max\{\frac{q_A v^2}{4}, q_A(v - \Lambda/W)\Lambda/W\} < \tilde{p}_j$ *for all* $j \in \{A, B\}$.

Note that the scenario where both SAs obtain information from the same ESC can not occur in an equilibrium path. *Thus, if the ESCs offer the same quality, then there will be no competition for the user market.* Figure 1 shows the profits of the two SAs as a function of $\tilde{p}_B$. For these parameters, when $\tilde{p}_B$ is small enough the equilibrium falls into case 2 in Theorem 8 with only SA 2 in the market. When $\tilde{p}_B$ is larger, the equilibrium falls into case 4 and neither SA will enter the market. The only scenario where both the SAs will be in the market is when they obtain information from different ESCs, which requires that both ESC offers different qualities of information. Figure 2 shows a scenario where both the SAs have positive profits when $\tilde{p}_B$ is small enough. Having such competition can provide positive consumer surplus. Note also that if two ESC are operating in the market, improving the quality of the poorer ESC may in fact hurt the SA profits or lead to a monopolistic scenario, in which one SA stays out of the market. The monopoly profit is higher compared to the competitive one. However, if there is a monopoly, it never covers the entire subscription base in contrast to the competitive outcome. *From a regulatory point-of-view, these results suggest their may be a benefit in encouraging ESCs that offer different service qualities.*

Note that when W is small, only the 4th case of Theorem 8 is satisfied. Hence, neither SAs will obtain information from the ESCs. On the other hand if W is large, Corollary 1 shows that only one of the SAs may obtain the information from the ESCs. If $W \to \infty$ and $q_j v \Lambda \geq \tilde{p}_j$ then only one of the SAs will obtain information from one of the ESCs in an equilibrium path.

IV. Heterogeneous Users and Unlicensed Access

In this section, we generalize our analysis to a scenario where the users are heterogeneous. Specifically, we consider a setting where each user's valuation v is drawn from a continuous distribution function $F(\cdot)$ so that $1 - F(v)$ denotes the mass of users who have valuations more than v. Throughout this section, we assume that $g(\cdot)$ is linear as in Assumption 1.

A. User Equilibrium

With heterogeneous users, each user will still select the SA that gives it the largest expected pay-off, where the user pay-offs are determined as in the previous section. The key difference now is that the Wardrop equilibrium conditions need to be modified to account for the heterogeneous valuations. Namely, if a user is not served by any SA, then the expected pay-off of that user must be less than or equal to zero and thus so must the expected pay-off of any user with a lower valuation. The following lemma shows how to account for this in a user equilibrium in which both SA's serve customers and acquire information from different ESCs.

Lemma 1. *When SA 1 acquires information from ESC A and SA 2 acquires information from ESC B, then if there is a user equilibrium, $(\lambda_1^*, \lambda_2^*)$ in which both SA's serve customers it must* *satisfy the following*

$$\lambda_1^* = 1 - F(\lambda_1^*/W - \frac{p_1 - p_2}{q_A - q_B}),$$
$$\lambda_2^* = F(\lambda_1^*/W - \frac{p_1 - p_2}{q_A - q_B}) - F(\lambda_1^*/W + \lambda_2^*/W - \frac{p_2}{q_B}). \tag{20}$$

Likewise, if only one of the SAs i offers service, the user equilibrium is given by

$$\lambda_i^* = 1 - F(\lambda_i^*/W - \frac{p_i}{q_j}) \tag{21}$$

where SA i obtains information from ESC j $j \in \{A, B\}$. Similar expressions can be derived for the case where both SA's acquire information from the same ESC.

B. Market Equilibrium

Now, we describe the overall market equilibrium.

Theorem 9. *The market equilibrium exists and must be one of the following:*

1) *Only one of the SAs obtains information from ESC A.*
2) *Only one of the SAs obtains information from ESC B.*
3) *Both the SAs obtain information from different ESCs.*
4) *Neither of the SAs obtains information from the ESCs.*

Further in any such equilibrium, the entire market is not served.

As in the homogeneous case, both SAs never obtain information from the same ESC. However, in contrast to the homogeneous case, now the SAs never cover the whole market as there will be some users whose valuation will be low enough that they are not served.

V. Licensed Spectrum

Next, we consider the scenario where the SAs each have licensed access to a portion of the shared spectrum similar to the Priority Access (PA) tier in the CBRS system. In this case, whenever the spectrum is available, the SA can exclusively use its licensed portion. To determine availability, the SAs still need to obtain information from an ESC.

A. Model with licensed sharing

We again consider that the users are homogeneous, and, thus, have the same valuation v. The total bandwidth W is split equally among the two SAs. Thus, each SA has $W/2$ amount of bandwidth. If a user chooses to get service from one of the SAs, in this case that user only faces congestion due to the other customers of that SA (and not from the customers of the other SA). Hence, in this case we model the congestion cost for customers of SP i as $g(\frac{\lambda_i}{(W/2)})$ and again assume that this cost is linear as in Assumption 1. Thus, if SA i obtains information from the ESC j, the customers of SA i will have a pay-off of

$$q_j v - p_i - q_j \frac{\lambda_i}{W/2}. \tag{22}$$

Note in this case the pay-off only depends on the probability q_j associated with the ESC that SA i obtains information from and, moreover, will be the same regardless of the ESC selection decision of the other SA. The profit expressions of the SAs and the rest of the model remain the same as in the unlicensed case. We next characterize the sub-game perfect equilibrium by backward induction, beginning with the user equilibrium.

B. User Equilibrium

Again, given the prices selected by the SAs and their ESC choices, the user equilibrium can be characterized as a Wardop equilibrium as in Section II-C. We next use this characterization to specify the user equilibrium under different ESC selection choices. First, we consider the case where both SAs obtain information from the same ESC. Note that unlike with unlicensed access, with licensed access, the two SAs will have *different* congestion costs whenever they serve different numbers of users.

Theorem 10. *Assume that both SAs obtain information from ESC j ($j \in \{1, 2\}$) and that for each SA i, $p_i \leq q_j v$,*

1) *If $p_1 = p_2$ and $q_j v - q_j \lambda/W - p_1 \geq 0$, then $\lambda_1 = \lambda_2 = \Lambda/2$.*
2) *If $p_1 = p_2$ and $q_j v - q_j 2\alpha/W - p_1 = 0$ for some $\alpha < \Lambda/2$, then $\lambda_1 = \lambda_2 = \alpha$.*
3) *If $p_i \neq p_k$ (for $i \neq k$), then the unique equilibrium is $q_j v - q_j 2\lambda_1/W - p_k = q_j v - q_j 2\lambda_2/W - p_i$, where $0 \geq \lambda_1 \leq \Lambda$, and $0 \geq \lambda_2 < \lambda_1$.*

If the condition $p_i \leq q_j v$ is not satisfied, then it can be seen that in equilibrium SA i will never attract any customers and so the resulting user equilibrium is the same as if that SA did not enter the market. When this condition is strictly satisfied, that SA will always be able to attract some customers in equilibrium. Note also that if $p_1 = p_2$, then unlike with unlicensed spectrum, the user equilibrium is unique and equally divided. If one of the SAs sets a higher price, i.e., $p_i > p_j$, unlike the unlicensed case, both the SAs may have a non-zero user base. If the total number of customers served is less than Λ, then fixing the prices, the market coverage (given by the parameter α) will again be higher if the SAs obtain information from ESC A rather from ESC B.

Next consider the monopolistic scenerio in which one SA obtains information from an ESC and the other does not. In this case, the situation is essentially the same as in unlicensed case except that the SA obtaining information only has a bandwidth of $W/2$ instead of W. Hence, the user equilibrium is again given by case 3 and 4 in Theorem 1 with W replaced by $W/2$ (where SA k is the monopolist SA).

Finally, consider when SA 1 and 2 obtain information from different ESCs. Without loss of generality, we again assume that SA 1 (2) obtains information from ESC A (B).

Theorem 11. *Assume SA* 1 *(*2*) obtains information from ESC* A *(*B*), the* unique *user equilibrium* (λ_1, λ_2) *satisfies:*

1) $\lambda_1 = \lambda_2 = 0$*, if* $q_A v - p_1 < 0$*, and* $q_B v - p_2 < 0$*;*
2) $\lambda_1 = \min\{\alpha, \Lambda\}$*, and* $\lambda_2 = 0$ *if* $q_A v - q_A 2\lambda_1/W - p_1 \geq q_B v - p_2$ *where* α *satisfies* $q_A v - q_A 2\alpha/W - p_1 = 0$.
3) $\lambda_2 = \min\{\alpha, \Lambda\}$*, and* $\lambda_1 = 0$*, if* $q_B v - q_B 2\lambda_2/W - p_2 \geq q_A v - p_1$ *where* α *satisfies* $q_A v - q_A 2\alpha/W - p_1 = 0$.
4) $\lambda_1 > 0, \lambda_2 > 0$ *such that* $q_A v - q_A 2\lambda_1/W - p_1 = q_B v - q_B 2\lambda_2/W - p_2$*; and* $q_A v - q_A 2\lambda_1/W - p_1 \geq 0$*, and* $\lambda_1 + \lambda_2 \leq \Lambda$.

In the first case in Theorem 11, users do not subscribe to any of the SAs. Similar to the discussion following Theorem 10, this is because their prices are two high to attract any customers. In the second case, the subscribers only subscribe to SA 1 as the expected payoff attained by the users is positive for SA 1 and strictly greater than that of SA 2 even when SA 2 has no congestion. Case 3 is the corresponding result when only SA 2 serves customers. In case 4, both SAs serve the market. However, they may or may not serve the entire market. This will depend on the prices p_1, p_2 and the probabilities q_A, q_B and the valuation v. The split of the market is unique. In this unique split, as the quality of information from ESC A (i.e., q_A)

increases, the market share of SA 2 will decrease. On the other hand, if p_i increases, the market share of SA i will decrease.

C. Price Equilibrium

Next we turn to the second stage. Recall, in this stage, given the ESC choices, each SA i selects its service price p_i to the revenue $p_i\lambda_i$. When doing this, λ_i will be specified by the corresponding user equilibrium determined in the previous section, which in turn depends on if the SAs obtain information from the same ESC or a different ESC, or if one SA does not obtain information. We treat each of these cases separately.

1) Both SAs obtain information from the same ESC: First, we describe the equilibrium pricing strategy when both the SAs obtain information from the same ESC. We further divide this into three cases depending on the relationship of v, Λ and W.

Theorem 12. *If both SAs obtain information from ESC j and if $v \geq 3\Lambda/W$, the second stage pricing strategy is*

$$p_i^* = 2q_j\Lambda/W \tag{23}$$

for $i = 1, 2$. The third-stage user equilibrium is

$$\lambda_1 = \lambda_2 = \Lambda/2. \tag{24}$$

The profits of the SAs are

$$q_j\Lambda^2/W - \tilde{p}_j. \tag{25}$$

Note that the traffic is equally split among the SA's as the prices selected by the SAs are the same. Also note that the price is higher if the SAs obtain information from the ESC A rather than B. Even though $q_A\Lambda^2/2 > q_B\Lambda^2/2$, $\tilde{p}_A$ may be higher than $\tilde{p}_B$. Thus, it is not clear whether the SAs will attain a higher payoff if they obtain information from ESC A in the first stage. The user's surplus is strictly positive in this scenario.

Note that the condition $v \geq 3\Lambda/W$ is more likely to be satisfied when W is large. However, from (25)) the profits of the SAs decrease as W increases. Thus, the profits may be negative for large enough W. Later, we will show that in the equilibrium path, this is not sustainable when W is very large. Thus, as in the unlicensed case, there may not be any competition when W is large. The profits decrease as W increases due to the fact that the prices also decrease with the increase in W. Intuitively, as W increases, the market becomes more competitive which drives down the prices, eventually leading to an SA leaving the market.

Theorem 13. *If both SAs obtain information from ESC j and $2\Lambda/W \leq v \leq 3\Lambda/W$, the second stage pricing strategy is*

$$p_i^* = q_j(v - \Lambda/W) \tag{26}$$

for $i = 1, 2$. The third-stage user equilibrium is

$$\lambda_1 = \lambda_2 = \Lambda/2. \tag{27}$$

The payoffs of the SAs are

$$q_j(v - \Lambda/W)\Lambda/2 - \tilde{p}_j. \tag{28}$$

Similar to Theorem 12, in Theorem 13, each SA covers the half of the user base. However, unlike Theorem 12, the user's surplus is 0. The price is higher if SAs obtain information from ESC A rather than B. However, the payoff will again depend on $\tilde{p}_j$s.

Note that when W is large, $3\Lambda/W$ is very small. Thus, the upper bound on v stated in this theorem is less likely to be satisfied. Similarly, when W is small, $2\Lambda/W$ is large making the lower bound on v less likely to hold. Also note that the prices and profits of the SAs increase as W increases unlike Theorem 12.

Finally, we show the equilibrium when $v \leq \Lambda$.

Theorem 14. *If both SAs obtain information from ESC j and $v \leq 2\Lambda/W$, the second stage pricing strategy is*

$$p_i^* = q_j v/2 \tag{29}$$

for $i = 1, 2$. The third-stage user equilibrium is

$$\lambda_1 = \lambda_2 = Wv/4. \tag{30}$$

The profits of the SAs are

$$Wq_j v^2/8 - \tilde{p}_j. \tag{31}$$

Similar to Theorems 12, and 13, in Theorem 14, the traffic is equally split. However, it does not cover the whole user base unlike Theorems 12, and 13. The user surplus is again 0 similar to Theorem 13) but unlike unlike Theorem 12. The price is higher if the SAs obtain information from the ESC A rather than B but the pay-off will again depend on $\tilde{p}_j$.

As W increases the profits of SAs increase since the demand increases with W. However, the prices are independent of W. Note that when W is small, $2\Lambda/W$ is large ,making the bound on v hold for a larger range of v. However, when W is small, the profits may be negative, which will not be sustainable on the equilibrium path.

2) Monopoly scenario: Next, we consider the scenario where only one of the SAs obtains information from an ESC and so is essentially a monopolist when making its pricing decision. As noted in the previous section, this is exactly the same as in Theorem 4 with $W/2$ in place of W.

3) The SAs obtain information from different ESCs: Finally, we consider the price equilibrium when the SAs obtain information from different ESCs. Later, we will show that with licensed sharing such an equilibrium is not sustainable on the equilibrium path. Again we divide this into two cases depending in part on the user valuation v.

Theorem 15. *Under Assumption 1, assume that SA* 1 *(*2*) obtains information from ESC* A *(*B*). If*

$$v \geq \frac{5q_Aq_B + 2q_B^2 + 2q_A^2}{q_A^2 + 4q_Aq_B + q_B^2}(2\Lambda/W) \tag{32}$$

then in the unique price equilibrium (p_1^*, p_2^*) *is given by*

$$\begin{aligned} p_1^* &= \frac{4q_B\Lambda/W + 2q_A\Lambda/W + 2(q_A - q_B)v}{3} \\ p_2^* &= \frac{4q_A\Lambda/W + 2q_B\Lambda/W - 2(q_A - q_B)v}{3}. \end{aligned} \tag{33}$$

The corresponding user equilibrium (third-stage) is given by

$$\begin{aligned} \lambda_1^* &= \frac{2q_B\Lambda + q_A\Lambda + (q_A - q_B)v(W/2)}{3(q_A + q_B)} \\ \lambda_2^* &= .\frac{2q_A\Lambda + q_B\Lambda - (q_A - q_B)v(W/2)}{3(q_A + q_B)}. \end{aligned} \tag{34}$$

The profits of the SAs' are respectively

$$\begin{aligned} \pi_1^* &= \frac{2(q_A + q_B)}{W}(\lambda_1^*)^2 - \tilde{p}_A. \\ \pi_2 &= W\frac{2(q_A + q_B)}{W}(\lambda_2^*)^2 - \tilde{p}_B. \end{aligned} \tag{35}$$

The condition in (32) implies that the market share of SA 1 is higher than SA 2. The first term in the profit of SA 1 is also strictly higher compared to SA 2. However, SA 1's profit may be lower compared to that of SA 2 due to the payment $\tilde{p}_A$. Also note that as the difference between q_A and q_B decreases, the profits of the SA 2 becomes closer to SA 1. When $q_B = q_A$, note that the condition in (32) becomes the same as that in Theorem 12 and prices and quantities are also equal to those in that theorem. Different from the unlicensed case, here equalizing the quantities does not necessarily lead to a negative profit for the SAs. The sum of λ_1 and λ_2 in the equilibrium is equal to the total number of subscribers. Hence, when the SAs obtain information from different ESCs and the condition in (10) is satisfied, the SAs select prices such that they cover the entire subscription base.

The condition in (32) is clearly satisfied when W is very large. Though the profit of SA 1 increases with W, the profit of SA 2 decreases with W. Intuitively, when W is large, SA 1 can select lower prices and serve a large number of users. Hence, SA 2 suffers because of the inferior quality of information.

Next we characterize a price equilibrium when condition (32) is not satisfied.

Theorem 16. *Under Assumption 1, assume that SA 1 (2) obtains information from ESC A (B). If*

$$3\Lambda/W < v < \frac{5q_Aq_B + 2q_A^2 + 2q_B^2}{q_A^2 + 4q_Aq_B + q_B^2}(2\Lambda/W) \tag{36}$$

then in the unique price equilibrium (p_1^*, p_2^*) is given by

$$\begin{aligned} p_1^* &= q_A(v - 2\Lambda/W) \\ p_2^* &= q_B(4\Lambda/W - v). \end{aligned} \tag{37}$$

The corresponding user equilibrium is

$$\begin{aligned} \lambda_1 &= Wv/2 - \Lambda. \\ \lambda_2 &= 2\Lambda - Wv/2. \end{aligned} \tag{38}$$

The profits of the SA's are

$$\begin{aligned} \pi_1 &= \tfrac{W}{2}q_A(v - 2\Lambda/W)^2 - \tilde{p}_A \\ \pi_2 &= \tfrac{W}{2}q_B(4\Lambda/W - v)^2 - \tilde{p}_B. \end{aligned} \tag{39}$$

Similar to Theorem 15, the total market share of the SAs cover the whole subscription base Λ. The price set by SA 1 is higher compared to SA 2. However, the consumer surplus is zero unlike in Theorem 15. The market share of SA 1 is higher compared to the SA 2. The payoffs of the SAs are also lower compared to Theorem 15. This is because Theorem 16 is valid when v is smaller compared to Theorem 15. Note in this case, when $q_A = q_B$, the condition in (32) will not hold and so this case does not arise when the ESCs offer the same quality. The impact of W on the profits and the prices of SAs are similar to Theorem 15.

Finally we look at the case where v is sufficiently small.

Theorem 17. *Under Assumption 1, assume that SA 1 (2) obtains information from ESC A (B). If $v \leq 3\Lambda/W$, then the unique price equilibrium (p_1^*, p_2^*) is given by*

$$\begin{aligned} p_1^* &= q_A \min\{v/2, v - \Lambda/W\} \\ p_2^* &= q_B \min\{v/2, v - \Lambda/W\}. \end{aligned} \tag{40}$$

The corresponding user equilibrium is

$$\lambda_1^* = \lambda_2^* = W/2 \min\{v/2, v - \Lambda/W\}, \tag{41}$$

and the SAs' profits are

$$\begin{aligned} \pi_1 &= q_A W/2 \min\{v/2, v - \Lambda/W\}\lambda_1^* - \tilde{p}_A \\ \pi_2 &= q_B W/2 \min\{v/2, v - \Lambda/W\}\lambda_2^* - \tilde{p}_B. \end{aligned} \tag{42}$$

In this equilibrium, the subscribers are again split among the two SAs. However, in contrast to Theorems 15 and 16, in Theorem 17 the number of subscribers are the same for each SA. When $v < \Lambda$, there will be some users who will not subscribe to any of the SAs. However, when $v \geq \Lambda$, the total market share of the SAs is equal to the total number of users Λ. Note that the equilibrium expression is similar to the scenario where the SAs obtain information from the same ESC. The only difference is that the price and payoff of the SAs 1 and 2 are multiplied by q_A and q_B rather than the same parameter (either q_A or q_B).

Similar to Theorem 15, in Theorem 17 the price set by SA 1 is higher compared to SA 2. The profits of the SAs are lower compared to that obtained in Theorems 15 and 16 since the above result holds for smaller values of v. In this scenario, the profits and prices decrease with the increase in W.

D. ESC Selection Equilibrium

We now turn to the first stage and specify the ESC selection equilibrium. First, we introduce some notation that will be useful to specify the equilibrium profits of the SA's. Specifically, define r_j for each ESC j as

$$r_j = \begin{cases} q_j v^2/8 - \tilde{p}_j, \quad \text{if } v < 2\Lambda/W, \\ q_j(v - \Lambda/W)\Lambda/2 - \tilde{p}_j, \quad \text{if } 2\Lambda/W \leq v \leq 3\Lambda/W, \\ q_j \Lambda^2/W - \tilde{p}_j, \quad \text{if } v > 3\Lambda/W. \end{cases}$$

Theorem 18. *In this first stage under Assumption 1 only one of the following five equilibria are possible:*

1) *Both the SAs obtain information from ESC A if $r_A \geq 0$ and $r_A \geq r_B$.*
2) *Both the SAs obtain information from the ESC B a if $r_B \geq 0$ and $r_B \geq r_A$.*
3) *One of the SAs obtains information from the ESC A if $r_A < 0$, $r_B < 0$, and*
$$W/2 \max\{\frac{q_A v^2}{4}, q_A(v - 2\Lambda/W)2\Lambda/W\} - \tilde{p}_A \geq 0.$$
4) *One of the SAs obtains information from the ESC B if $r_A < 0$, $r_B < 0$, and*
$$W/2 \max\{\frac{q_B v^2}{4}, q_B(v - 2\Lambda/W)2\Lambda/W\} - \tilde{p}_B \geq 0.$$
5) *Neither SA obtains information from any of the ESCs if*
$$\max\{\frac{W q_B v}{4} \min\{v/2, \Lambda\}, q_B(v - \Lambda)\Lambda\} - \tilde{p}_B < 0$$
and
$$\max\{\frac{W q_A v}{2} \min\{v/2, \Lambda\}, q_A(v - \Lambda)\Lambda\} - \tilde{p}_A < 0.$$

Thus, in contrast to Theorem 8, with licensed spectrum, there can be an equilibrium in which both the SAs obtain information from the same ESC. Unlike the unlicensed spectrum, there is no equilibrium where the SAs obtain information from different ESCs. Thus, only one ESC can exist in the market if the spectrum is licensed. Hence, licensed spectrum does not lead to a competition among the ESCs unlike the unlicensed case. However, multiple SAs with the same quality can co-exist with licensed access but not with unlicensed. Similar to the unlicensed case, there can be scenarios in which only one SA exists in the market (cases 3 and 4) or in which no SA will find it profitable to enter (case 5). When only one SA exists in the market, the corresponding price equilibrium is as in Theorem 4 with W replaced by $W/2$.

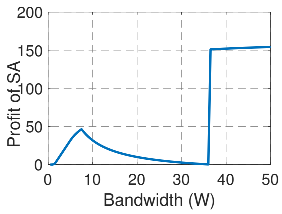

Fig. 3. Variation of the profits of the SAs with bandwidth (W) for licensed access with parameters $v = 10, \tilde{p}_A = 10, \tilde{p}_B = 3, q_A = 0.7, q_B = 0.5, \Lambda = 25$.

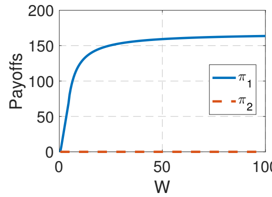

Fig. 4. Variation of the profits of the SAs with bandwidth (W) for unlicensed access with the same parameter as in Fig. 3.

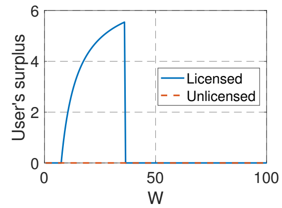

Fig. 5. Variation of users' surplus with W both for licensed and unlicensed spectrum with the same parameters as in Fig. 3.

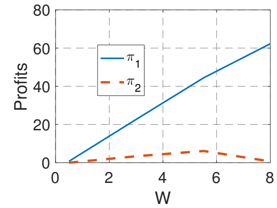

Fig. 6. Variation of the profits of SAs with W for unlicensed spectrum where $v = 10, \Lambda = 50, \tilde{p}_A = 10, \tilde{p}_B = 3, q_A = 0.7, q_B = 0.5$.

When both SA are in the market (cases 1 and 2), the corresponding price equilibrium depends on how v compares to v compares Λ/W as in Theorems 15-17. In each of these cases, both SAs serve the same number of users at the same price. The competition between the SA can generate positive consumer surplus, but only when the user valuation is sufficiently high, namely when $v > 3\Lambda/W$.

Our analysis reveals that the SAs acquire information from the same ESC. This insight also carries over to multiple ESCs, i.e., even if there are k ESCs, the SAs will acquire information from the same ESC in the first stage. Thus, only one ESC can exist.

In the following we further characterize the impact of W on the first stage equilibrium.

Corollary 2. *When $W \geq \dfrac{q_j v - \tilde{p}_j}{2 q_j \Lambda^2}$, there is no equilibrium where both the SAs obtain information from an ESC. When $W \leq \dfrac{8\tilde{p}_j}{q_j v^2}$, neither SA obtains information from an ESC.*

This shows that vary W has a similar impact on the first stage equilibrium as in the unlicensed case. When W is large, the competitive equilibrium where both the SAs will serve users is not sustainable similar to the unlicensed case. However, when W is large enough, the monopolistic scenario may arise where only one of the SAs will obtain information from one of the ESCs. Similarly, when W is small, the profits of the SAs are very low even in the monopolistic scenario. Thus, neither SA obtains information from an ESC.

E. Numerical Examples

In this section we provide some numerical examples to illustrate the previous results and to compare the licensed and unlicensed cases. First, in Fig. 3 we show the variation of the profit of the SAs with licensed access as a function of W. Note that when W is small ($W \leq \frac{8\tilde{p}_j}{q_j v^2}$), the profits are 0 as neither SA obtains information from the ESCs. When W increases both the SAs obtain information from the same ESC. The profits of both SAs intitially increase with W and are identical. However, as W increases the competition between the SAs increases eventually causing profits to decrease. When

$$W = \frac{q_j v - \tilde{p}_j}{2q_j\Lambda^2}$$

both SAs earn 0 profit. When W increases beyond that point, only one of the SAs obtain information from ESC A, the other SA does not offer any service. The profit of the SA that stays in the market becomes equal to the monopoly profit.

For the same parameters, Fig. 4 shows the variation of the SAs' profits with unlicensed access as a function of bandwidth (W). Note that unlike the licensed case, in the unlicensed case at most one of the SAs (SA 1) obtains information from the ESCs. The payoff of the SA 2 is thus zero. Though initially the profit of SA 1 increases rapidly with the increase in W, the increment becomes small when W exceeds a threshold. When W is small none of the SAs obtain information from the ESCs. Hence, the profits of both the SAs are zero.

Fig. 5 shows the variation of users' surplus with W for the licensed and unlicensed spectrum (i.e., the total pay-off of all users). In the unlicensed case, since only one of the SAs enters the market, the users' surplus is zero (i.e. the monopolist SA that enters extracts all surplus). In the licensed case, when W is small, the users' surplus is zero. However, as W increases, the surplus becomes positive once $v \geq 3\Lambda/W$. However, if W exceeds a threshold (40), only one of the SAs offers service, thus, the surplus again becomes negative.

Comparing the plots we see that when W is not too large, the unlicensed case generates more profit as it prevents competition for the given parameters. However, the licensed case can generate positive consumer surplus when W is small enough, while in the unlicensed case the surplus will always be zero.

Fig. 6 shows an unlicensed scenario where both the SAs serve users when $W \leq 8$ for a different set of parameters compared to those in Fig. 4. Unlike the scenario in Fig. 4, competition among the SAs exists when $W \leq 8$. However, when $W > 8$ only SA 1 enters the market. Note when this occurs, SA 1 profit increases much more quickly with W due to the lack of competition.

VI. Conclusion

We have considered a simple model of markets for spectrum measurements motivated by the the CBRS system. A key feature of our model is that firms offering wireless service must acquire information about spectrum availability from an ESC, where different ESCs may offer different qualities of information. Our results show that the impact of such differences in information quality depend strongly on the underlying licensing paradigm used. With unlicensed access, different information qualities are needed to promote competition, while with licensed access, different information qualities can not be supported in equilibrium.

There are many directions this work could be extended including considering more SAs or ESCs in the market. We have considered models where the spectrum is either entirely licensed or unlicensed. A hybrid model in which portions of the spectrum are licensed and unlicensed is a possible extension. We focused on the price competition among SA's and assumed that the prices for spectrum measurements was given. Characterization of those prices is also of interest.

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