Efficient Power Allocations in Wireless ARQ Protocols

Naveen Arulselvan and Randall Berry
Department of Electrical and Computer engineering
Northwestern University, Evanston, IL, 60201.
{naveen,rberry}@ece.nwu.edu

Abstract

In this paper, efficient power allocation strategies for ARQ protocols operating in wireless environments are studied. Power as well as transmission rate is optimally adapted by the sender based on channel state information (CSI) obtained through feedback, while guaranteeing quality-of-service (QOS) constraints such as average throughput or average delay. The power policies adopted for basic ARQ protocols are characterized, and a comparison of their performance is studied. Numerical results are presented for a Rayleigh fading channel.

Keywords

Wireless Networks, Retransmission protocols, error control, power control

INTRODUCTION

Power conservation is a key concern for wireless technologies. Protocols deployed in wireless networks must judiciously utilize the available power as well as deliver reliable communications. In this work, we study power efficiency in the context of link-level error control for a wireless network. Specifically, we consider various automatic repeat request (ARQ) protocols, which may be used to provide reliability to delay-tolerant applications. For these protocols, we investigate the trade-offs between the average transmitted power and various QoS parameters such as average throughput and delay. We also present a comparison between the performance of Stop-and-Wait and Go-Back-N protocols.

There is a growing awareness that a stronger coupling between traditional network layers can be beneficial in wireless settings [2]. In this work, we explore such a coupling between the physical layer and link-layer retransmission protocols. We focus on the performance of different ARQ protocols when physical layer parameters, such as transmission rate and power can be adapted, based in part on the available CSI. Here, CSI could be the exact or average fade level or any other meaningful description of the physical layer. Several other approaches along these lines have been investigated in the literature. In [7],[8],[9], transmission policies to minimize the average energy expended are studied. An adaptive algorithm to optimally adjust the packet lengths is examined in [4].

CHANNEL MODEL

We consider the situation shown in Fig. 1, in which the sender transmits packets over a narrow-band block-fading

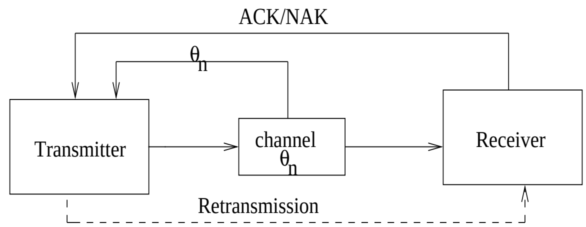

Figure 1. Feedback based error control

channel with additive noise. To simplify our discussion, we assume the block length is equal to the packet transmission time. A discrete-time model is used, where each time unit corresponds to the block length. After every time step, the transmitter receives new CSI, independent of whether a transmission attempt was made or not. We assume this CSI takes values from the finite set $\Theta = \{\theta_1, \ldots, \theta_M\}$. The actual fade level of the channel depends conditionally on the current CSI. The channel is assumed to have a fixed propagation delay, I, after which the receiver acknowledges each packet as correctly received or in error. If in error, the packet is retransmitted according to the ARQ protocol.

Initially, we assume that the transmission rate is fixed at R bits per second, and that the transmitter can adapt the transmission power used to send each packet. Let P_i denote the power allocated when the CSI has value θ_i. The probability a packet arrives in error depends on the transmitted power P_i and the available CSI, θ_i. We express this via a function $\rho(P_i, \theta_i)$. This function is assumed to be decreasing in P_i; a specific example is given in the next section. A case where the sequence of CSI values are modelled as a stationary, ergodic Markov chain is considered. Let $p_\theta(\cdot)$ denote the steady-state probability distribution of $\{\theta_i\}$. The steady-state average probability of a successful transmission is given by

$$q = \sum_{i=1}^{M} (1 - \rho(P_i, \theta_i))\, p_\theta(\theta_i). \tag{1}$$

Similarly, the average power $\bar{P}$ expended is found to be,

$$\bar{P} = \sum_{i=1}^{M} P_i p_\theta(\theta_i). \tag{2}$$

Problem Formulation

For most common ARQ protocols, the needed success probability q can be determined given a required average throughput of R'. For instance, consider a Go-Back-N protocol with large enough window size W_{sat} so that the transmitter never idles if packets are available [6]. In this case, the needed success probability is given by

$$q = \frac{W_{sat}R'}{\frac{DR}{F} + (W_{sat} - 1)R'}, \tag{3}$$

where D is the number of data bits in each packet, and F is the frame size. Given such a relation, the power allocation problem can be formulated as,

$$\begin{aligned} \min \quad & \bar{P} = \sum_{i=1}^{M} P_i p_\theta(\theta_i), \\ \text{s.t} \quad & \sum_{i=1}^{M} (1 - \rho(P_i, \theta_i)) p_\theta(\theta_i) = K_x(R'), \\ & P_i \geq 0, \quad \forall i, \end{aligned} \tag{4}$$

where $K_x(R')$ is a function that depends on the ARQ protocol employed. For instance, $K_{gb-n}(R') = q$ is given as in (3) for a Go-Back-N protocol. For a Stop-and-wait protocol [6],

$$K_{sw}(R') = \frac{R'(F + IR)}{DR}. \tag{5}$$

For a Selective Repeat protocol with large enough window size,

$$K_{SR}(R') = \frac{FR'}{DR}. \tag{6}$$

Let $P_x(R')$ denote the solution to (4) as a function of the required throughput R', for a given protocol. The function, $P_x(R')$ then describes the trade-off between power and throughput for that protocol. We note that the minimum energy per bit solution investigated in [9] can be interpreted as a particular value of $P_x(R')$.

RAYLEIGH FADING CHANNELS

We assume each packet is transmitted over a Rayleigh fading channel with Additive White Gaussian Noise (AWGN). The power spectral density of the noise process is N_0 and the bandwidth of the channel is B Hz. Let H_i be the channel fading when in the i^{th} channel state. We assume the CSI sequence $\{\theta_i\}$ provides the corresponding expected fade levels, i.e, $\theta_i = \mathbb{E}(H_i) = \bar{H}_i$, for all i. Let $\nu_i = \frac{P_i}{N_0 B}$ be the transmitted SNR when the current CSI is $\bar{H}_i$. We model the probability a packet is dropped using a "capacity vs. outage" framework [5]. Specifically, a packet is dropped if the Shannon capacity of channel realization during that particular block falls below the information rate R, this event being referred to as an outage. The outage probabilities in the M channel states can be calculated as,

$$\rho(\nu_i, \bar{H}_i) = 1 - \exp\left(-\frac{2^{\frac{R}{B}} - 1}{\nu_i \bar{H}_i}\right), \quad \forall \quad i. \tag{7}$$

Two State Channel

Consider a channel with two CSI states, with steady-state probabilities p_1 and p_2 and with expected gains $\bar{H}_1 > \bar{H}_2$. The power allocation problem in (4) can be formulated in terms of the transmission SNRs (equivalently powers) in each state. The optimal power policy is characterized below as a function of the average throughput requirement R'.

Proposition 1. *When $K_x(R') > p_1$, the transmission powers are*

$$\nu_1 = \frac{-(2^{\frac{R}{B}} - 1)}{\bar{H}_1 \ln t} \quad ; \quad \nu_2 = \frac{-(2^{\frac{R}{B}} - 1)}{\bar{H}_2 \ln\left(\frac{K_x(R') - p_1 t}{p_2}\right)}, \tag{8}$$

where t is a solution in

$$\frac{\bar{H}_2}{\bar{H}_1} = \frac{p_2 t \ln^2 t}{(K_x(R') - p_1 t) \ln^2\left(\frac{K_x(R') - p_1 t}{p_2}\right)}. \tag{9}$$

Proof: For $K_x(R') > p_1$, the power must be split in both the channel states to satisfy the constraint in (4). Using the constraints, ν_1 and ν_2 have the above form for some $t \in [0, 1]$. That t must satisfy (9) follows the first order optimality conditions for (4). ■

Proposition 2. *When $K_x(R') < p_1 e^{-1}$ the transmission powers are*

$$\nu_1 = \frac{-(2^{\frac{R}{B}} - 1)}{\bar{H}_1 \ln 2K_x(R')} \quad ; \quad \nu_2 = 0. \tag{10}$$

When $K_x(R') \in (p_1 e^{-1}, p_1)$, the power policy adopted is either as in (8) or (10).

When $K_x(R') < p_1 e^{-1}$, transmitting in both the channel states can be shown to be sub-optimal.

When $K_x(R') \in (p_1 e^{-1}, p_1)$, the power policy is either of (8) or (10). The exact nature of the solution seems to be specific to the problem.

Comparison of Stop-and-Wait and Go back N

We study the power required for a Go-Back-N and Stop-and-Wait protocol with M equally likely CSI values. The Go-Back-N protocol will transmit in W_{sat} slots before sliding it's window, where W_{sat} is the number of packets needed to 'fill the channel'. In this same time, a Stop-and-Wait protocol will have transmitted in one slot. Hence, we compare the total energy per W_{sat} slots, i.e. $P_{SW}(R')$ and $W_{sat} P_{gb-n}(R')$.

Proposition 3. *There exists a threshold $R'(W_{sat})$ such that,*

$$P_{SW}(R') < W_{sat} P_{gb-n}(R'), \ \forall \ R' < R'(W_{sat}). \tag{11}$$

Proof: In the power allocation problem for the Go-back-N protocol, if $W_{sat}\nu_i$ is replaced with ν_i, the equality constraint can be rewritten as,

$$\frac{1}{M} \sum_{i=1}^{M} \exp\left(-\frac{W_{sat}\left(2^{\frac{R}{B}} - 1\right)}{\nu_i \bar{H}_i}\right) = K_{gb-n}(R').$$

These are success probabilities in the various channel states and hence non-negative. Then, as a consequence of the multinomial theorem,

$$\frac{1}{M}\left(\sum_{i=1}^{M}\exp\left(-\frac{2^{\frac{R}{B}}-1}{\nu_i\bar{H}_i}\right)\right)^{W_{sat}} \geq K_{gb-n}(R').$$

The success probability function is monotonically increasing. Therefore, for Stop-and-Wait to be more power efficient than Go-Back-N, it is sufficient to have,

$$MK_{SW}(R') < (MK_{gb-n}(R'))^{\frac{1}{W_{sat}}}.$$

Substituting the protocol constants from (3) and (5), it follows that the inequality is true for some $R'(W_{sat}) \in (0, R)$. Therefore,

$$P_{SW}(R') < WP_{gb-n}(R'), \quad \forall R' < R'(W_{sat}). \quad \blacksquare$$

Thus, with a low enough power requirement, Stop-and-Wait will have a higher throughput than Go-Back-N. This can be contrasted with a wire-line network, in which Go-Back-N will always have a higher throughput. The reason for this is that Go-Back-N sends all packets in the current window, before sliding back. When the power is low and error rates are high, successive packets are thrown out, leading to a loss of energy. We note that as in a wireline network, a Selective Repeat protocol will always have a higher throughput than either Go-Back-N or Stop-and-Wait.

In Fig. 2, the trade-off between power and throughput for a Stop-and-Wait protocol, is compared with a Go-Back-N protocol employing a window size of 2. The link rate is 200 Kbps. In this case until about 30 percent throughput requirement, Stop-and-wait is more power efficient than Go-Back-N. As the throughput requirements increase, Stop-and-Wait proves to be less efficient due to the overhead involved in waiting for an ACK.

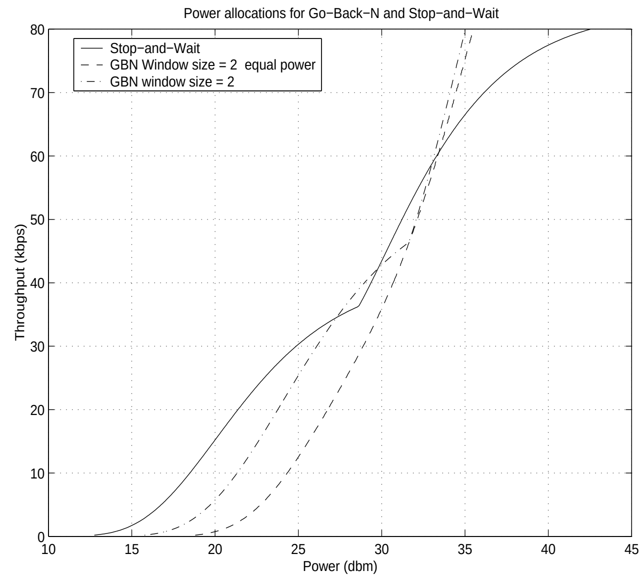

Figure 2. GBN vs SW - Rayleigh fading channel

Timesharing Strategy

The throughput versus power plot shown in Fig. 2 exhibits a distinct dip. This arises out of the nature of the success (complimentarily $error$) function, $S(\nu, \bar{H}_i) = 1 - \rho(\nu, \bar{H}_i)$, which is found to be convex for low SNR values and concave everywhere else. Over the region where $S(\nu, \bar{H}_i)$ is convex, throughput may be improved by using a timesharing scheme. In the timesharing scheme we suspend transmission with probability q, if the allotted transmission SNR falls below a threshold. Otherwise, the transmission SNR is fixed at the threshold value. Choosing the optimal timesharing scheme yields the modified success function:

$$\widetilde{S}(\nu, \bar{H}_i) = \begin{cases} \frac{\bar{H}_i e^{-1}\nu}{2^{\frac{R}{B}}-1}, & \text{if, } \nu < \frac{2^{\frac{R}{B}}-1}{\bar{H}_i} \\ S(\nu, \bar{H}_i), & \text{if } \nu > \frac{2^{\frac{R}{B}}-1}{\bar{H}_i}. \end{cases} \tag{12}$$

Figure 3 shows the power allocation curves for the modified as well as original success function, plotted on a linear scale. As expected, the average throughput obtained increases over a certain power range. Also notice, the throughput vs power curve is now concave throughout.

Rate Variation

In addition to power variation, we now assume that the transmitter adapts the number of information bits per packet, while keeping the keeping the transmission time of each packet the same. Therefore, both rate as well as power are adapted based on the CSI. We assume the maximum rate of transmission is R. Let R_i be the rate allotted when the CSI has value θ_i. To simplify our discussion, we consider transmission rates small compared to the channel bandwidth, (i.e),

$$2^{\frac{R}{B}} - 1 \approx \frac{R}{B}. \tag{13}$$

The analysis that ensues can be extended for higher rates of transmission as well. Using (13) in (7) we set,

$$\rho(P_i, R_i, \theta_i) = 1 - \exp\left(-\frac{R_i}{\nu_i \bar{H}_i B}\right), \quad \forall \quad i.$$

Note if $R_i = 0$, then clearly $\nu_i = 0$ and by convention $\rho(0, 0, \theta_i) = 1$. For a Stop-and-Wait protocol operating over a two state i.i.d. fading channel with gains $\bar{H}_1 > \bar{H}_2$, the power allocation problem in (4) can be modified with the protocol constant,

$$K_{rate,SW}(R') = \frac{T+I}{T}R',$$

This is expressed in terms of the packet transmission time, T. A similar constraint can be formulated for Go-back-N protocols as well.[1]

As an example, we consider a Stop-and-Wait protocol operating over a channel with $\bar{H}_1$ =1db and $\bar{H}_2$ =10db. The maximum transmission rate is set to 200 kbps. Figure 4 shows the power allocation scheme when packet sizes are

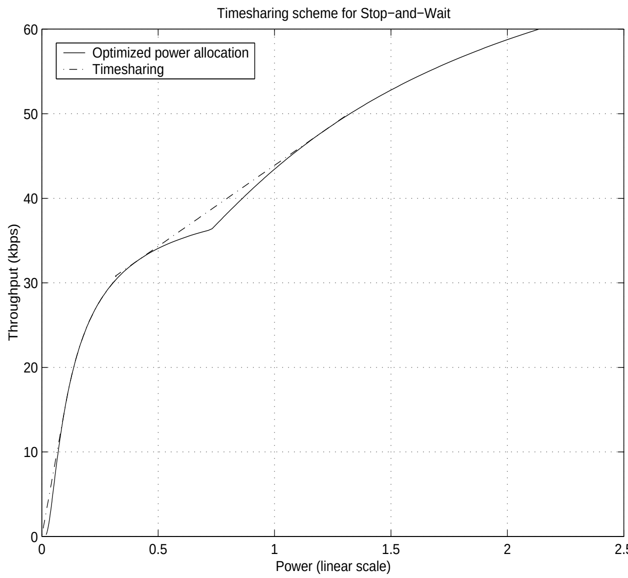

Figure 3. Timesharing

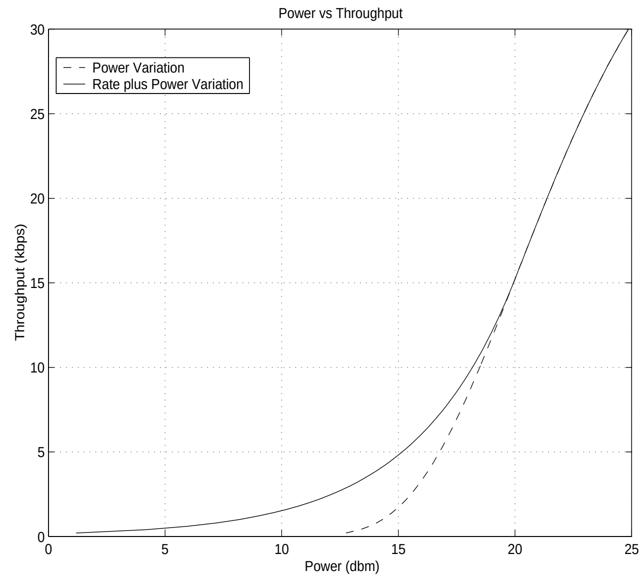

Figure 4. Optimal Rate plus Power adaptation

adjusted in addition to transmission power. The plot shows that in this case there is marginal benefit in adapting both rate and transmission power.

DELAY ANALYSIS

In some applications we may need to guarantee an average delay for each packet, while efficiently utilizing power. We study the case where packets arrive from a Poisson source at an average rate λ packets per time slot. Here each time slot is assumed to include the packet transmission time as well as round trip propagation delay, i.e one time slot equals $(\frac{F}{R}+I)$ seconds. The packets are buffered at the transmitter and constrained to have an average packet delay T_p. The packet losses are assumed to be i.i.d. The transmitter's queue then can be modelled as a M/G/1 queue with the service time distribution X given as $P\left(X=kW+1\right)=q(1-q)^k$, for all k [3]. So,

$$\bar{X}=1+\frac{W(1-q)}{q}, \tag{14}$$

$$\bar{X^2}=1+\frac{(W^2+2W)(1-q)}{q}. \tag{15}$$

Here q is the average probability of a successful transmission. The average packet delay, given by the Pollaczek-Khinchin formula is

$$T_p=\left(\bar{X}+\frac{\lambda\bar{X^2}}{2(1-\lambda\bar{X})}\right)\text{ (time slots).} \tag{16}$$

For Stop-and-Wait or Go-back-N operating over a Gaussian channel with two equally likely and independent states, the power allocation problem can be written as

$$\begin{aligned}\min\quad & \bar{\nu}=\frac{\nu_1+\nu_2}{2},\\ \text{s.t}\quad & \rho(\nu_1,h_1)+\rho(\nu_2,h_2)=2-2K_{del,x}(\lambda,T_p),\\ & \nu_1\geq 0,\quad \nu_2\geq 0.\end{aligned} \tag{17}$$

Notice that the structure of this problem is similar to (4). As in (4), we are restricting ourselves to power allocations that depend only on the channel state ; for the delay-constrained case we note that such a restriction is not optimal. Here $K_{del,x}(\lambda,T_p)$ is specific to the protocol employed and can again be interpreted as the average probability of a successful transmission. For a Go-Back-N protocol employing a window size W, using (14),(15) and (16) it can be shown that

$$K_{del,gbN}(\lambda,T_p)=\frac{\frac{2W}{\lambda}+W^2-2W+2WT_p}{\frac{2T_p}{\lambda}+(W-1)^2-2T_p(W-1)}. \tag{18}$$

Setting $W=1$, the protocol constant for Stop-and-Wait can be obtained. The solution form for a two state i.i.d. fading channel is as discussed for the throughput case.

Figure 5 gives the power policy adopted by the Stop-and-Wait protocol described in the previous sections, for various arrival rates. This is compared with an equal power splitting approach. Notice for small delay, an equal power is nearly optimal. There seems to be considerable benefit in employing this scheme, especially when the delay requirement is relaxed and arrival rates are low.

CONCLUSIONS

This paper discussed optimal power allocation schemes for ARQ protocols operating over fading channels and con-

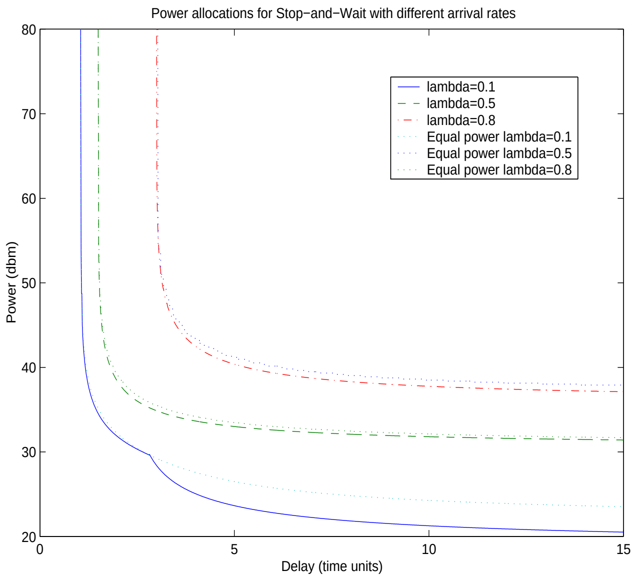

Figure 5. Effect of Arrival rate

strained by QOS requirements. The optimal power policy enhances performance as compared to a simple equal-power transmission scheme. The relative performance of the policies adopted by Stop-and-Wait and Go-Back-N protocols was also studied. It was shown that Stop-and-Wait is power efficient over a range of throughput requirements. This is an interesting result in the context of wireless channels, particularly because Stop-and-Wait is always less efficient in a wireline scenario. Finally, the power policy adopted while constrained by delay requirements was considered. Results similar to the throughput case were discussed.

REFERENCES

[1] N. Arulselvan. *Efficient Power Allocations in Wireless ARQ Protocols*. Masters project, Northwestern University, June 2000.

[2] E. Biglieri et.al. *Fading channels: information-theoretic and communication aspects*. IEEE trans. on IT, Vol. 44, No. 6, October 1998.

[3] R. Gallager and D. Bertsekas. *Data Networks*. Prentice-Hall, 1992.

[4] E. Modiano. *An adaptive algorithm for optimizing the packet size used in wireless ARQ protocols*. Wireless Networks, Vol.5, pp 279-286, 1999.

[5] L. Ozarow et.al. *Information theoretic considerations for cellular mobile radio*. IEEE trans. on Vehi. Tech., vol 43, pp 359-378, May 1994.

[6] J. Walrand. *Communication Networks*. McGraw Hill, $2^{nd} ed.$, 1998.

[7] D. Zhang and K. Wasserman. *Energy efficient data communication over fading channels*. WCNC, chicago, 2000.

[8] D. Zhang and K. Wasserman. *Transmission schemes for time-varying wireless channels with partial state observations*. INFOCOM, 2002.

[9] M. Zorzi and R. Rao. *Energy constrained error control for wireless channels*. IEEE Personal Comm., vol. 4, pp. 27-33, Dec. 1997.