

Recursion, this time in the language

```
{rec {<id>1 <FAE>1}  
    <FAE>2}
```

like **with** but **<id>₁** is bound in **<FAE>₂** and **<FAE>₁**

Defining Recursion

Last time:

```
{rec {<id>1 <FAE>1}  
      <FAE>2}
```

could be parsed the same as

```
{with {mk-rec ...mk-rec-code...}  
      {with {<id>1 {mk-rec {fun {<id>1}  
                          <FAE>1}}}}  
      <FAE>2}}
```

Defining Recursion

which is really

```
{ {fun {mk-rec}
  { {fun {<id>1} <FAE>2}
    {mk-rec {fun {<id>1}
              <FAE>1} } } } }
...mk-rec-code... }
```

Defining Recursion

Another approach:

```
(local [(define fac
         (lambda (n)
           (if (zero? n)
               1
               (* n (fac (- n 1))))))]
  (fac 10))
```

⇒

```
(let ([fac 42])
  (set! fac
        (lambda (n)
          (if (zero? n)
              1
              (* n (fac (- n 1))))))
  (fac 10))
```

Implementing Recursion

The **set!** approach to definition works only when the defined language includes **set!**.

But the **set!** approach to implementation requires only that the implementation language includes **set!**...

RCFAE Grammar

```
<RCFAE> ::= <num>
          | {+ <RCFAE> <RCFAE>}
          | {- <RCFAE> <RCFAE>}
          | <id>
          | {fun {<id>} <RCFAE>}
          | {<RCFAE> <RCFAE>}
          | {if0 <RCFAE> <RCFAE> <RCFAE>}
          | {rec {<id> <RCFAE>} <RCFAE>}
```



RCFAE Datatype

```
(define-type RCFAE
  [num (n number)]
  [add (lhs RCFAE?)
       (rhs RCFAE?)]
  [sub (lhs RCFAE?)
       (rhs RCFAE?)]
  [id (name symbol)]
  [fun (param symbol?)
       (body RCFAE?)]
  [app (fun-expr RCFAE?)
       (arg-expr RCFAE?)]
  [if0 (test-expr RCFAE?)
       (then-expr RCFAE?)
       (else-expr RCFAE?)]
  [rec (name symbol?)
       (named-expr RCFAE?)
       (body RCFAE?)])
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    [num (n) (numV n)]
    [add (l r) (num+ (interp l ds) (interp r ds))]
    [sub (l r) (num- (interp l ds) (interp r ds))]
    [id (name) (lookup name ds)]
    [fun (param body-expr)
         (closureV param body-expr ds)]
    [app (fun-expr arg-expr)
         (local [(define fun-val
                   (interp fun-expr ds))]
                 (interp (closureV-body fun-val)
                          (aSub (closureV-param fun-val)
                                (interp arg-expr ds)
                                (closureV-sc fun-val)))))]
    [if0 (test-expr then-expr else-expr)
         ...]
    [rec (bound-id named-expr body-expr)
         ...]))
```


RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    [num (n) (numV n)]
    [add (l r) (num+ (interp l ds) (interp r ds))]
    [sub (l r) (num- (interp l ds) (interp r ds))]
    [id (name) (lookup name ds)]
    [fun (param body-expr)
         (closureV param body-expr ds)]
    [app (fun-expr arg-expr)
         (local [(define fun-val
                   (interp fun-expr ds))]
                 (interp (closureV-body fun-val)
                         (aSub (closureV-param fun-val)
                               (interp arg-expr ds)
                               (closureV-sc fun-val)))))]
    [if0 (test-expr then-expr else-expr)
         ... (interp test-expr ds)
         ... (interp then-expr ds)
         ... (interp else-expr ds) ...]
    [rec (bound-id named-expr body-expr)
         ...]))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    [num (n) (numV n)]
    [add (l r) (num+ (interp l ds) (interp r ds))]
    [sub (l r) (num- (interp l ds) (interp r ds))]
    [id (name) (lookup name ds)]
    [fun (param body-expr)
         (closureV param body-expr ds)]
    [app (fun-expr arg-expr)
         (local [(define fun-val
                   (interp fun-expr ds))]
                 (interp (closureV-body fun-val)
                         (aSub (closureV-param fun-val)
                               (interp arg-expr ds)
                               (closureV-sc fun-val)))))]
    [if0 (test-expr then-expr else-expr)
         (if (numzero? (interp test-expr ds))
             (interp then-expr ds)
             (interp else-expr ds))]
    [rec (bound-id named-expr body-expr)
         ...]))
```

Testing For Zero

```
; numzero? : RCFAE-Value -> boolean  
(define (numzero? n)  
  (zero? (numV-n n)))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    ...
    [rec (bound-id named-expr body-expr)
      ...]))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    ...
    [rec (bound-id named-expr body-expr)
      ... (interp named-expr ds)
      ... (interp body-expr ds) ...]))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    ...
    [rec (bound-id named-expr body-expr)
      (local [(define new-ds (aRecSub bound-id
                                      ...
                                      ds))])
      ... (interp named-expr new-ds)
      ... (interp body-expr new-ds) ...))]))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    ...
    [rec (bound-id named-expr body-expr)
      (local [(define value-holder (box (numV 42)))]
        (define new-ds (aRecSub bound-id
                                value-holder
                                ds)))]
      ... (interp named-expr new-ds)
      ... (interp body-expr new-ds) ...))]))
```

RCFAE Interpreter

```
; interp : RCFAE DefrdSub -> RCFAE-Value
(define (interp a-rcfae ds)
  (type-case RCFAE a-rcfae
    ...
    [rec (bound-id named-expr body-expr)
      (local [(define value-holder (box (numV 42)))]
        (define new-ds (aRecSub bound-id
                                value-holder
                                ds)))]
      (begin
        (set-box! value-holder (interp named-expr new-ds))
        (interp body-expr new-ds))))))
```


RCFAE DefrdSub

```
(define-type DefrdSub
  [mtSub]
  [aSub (name symbol?)
        (value RCFAE-Value?)
        (sc DefrdSub?) ]
  [aRecSub (name symbol?)
           (value-box (box/c RCFAE-Value?))
           (sc DefrdSub?) ]])

(define-type RCFAE-Value
  [numV (n number?) ]
  [closureV (param symbol?)
            (body RCFAE?)
            (sc DefrdSub?) ]])
```

RCFAE Lookup

```
; lookup : symbol DefrdSub -> num
(define (lookup name ds)
  (type-case DefrdSub ds
    [mtSub () (error 'lookup "free variable")]
    [aSub (sub-name val rest-sc)
      (if (symbol=? sub-name name)
          val
          (lookup name rest-sc))]
    [aRecSub (sub-name val-box rest-sc)
      (if (symbol=? sub-name name)
          (unbox val-box)
          (lookup name rest-sc))]))
```