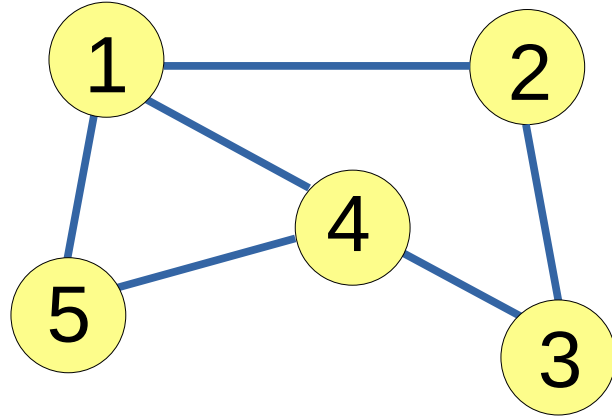# Reduction Example

H = 3-SAT   **N =** INDEPENDENT-SET

INDEPENDENT-SET

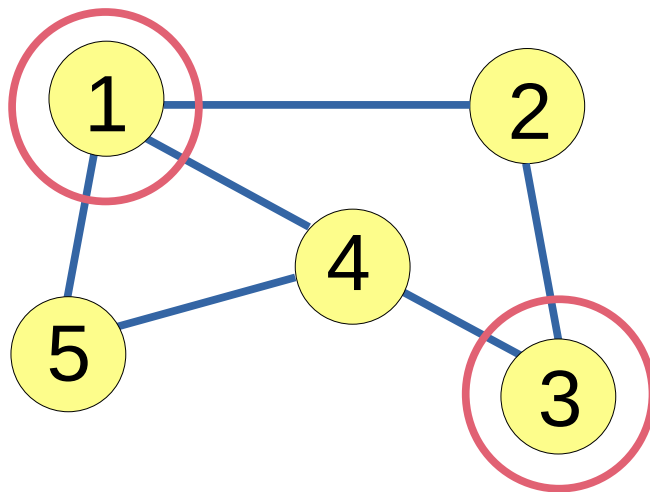Exists a set of k vertices s.t. no two are neighbors of each other?

k=2

Instance: a graph G and a threshold number k

Independent-Set

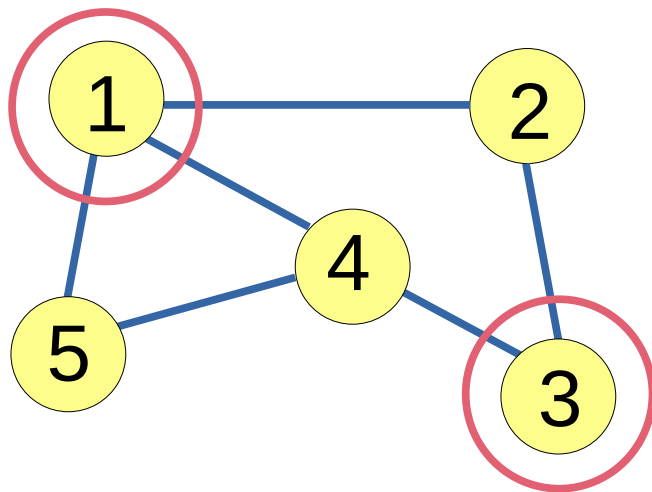Exists a set of k vertices s.t. no two are neighbors of each other?

k=2

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

Independent-Set

Exists a set of k vertices s.t. no two are neighbors of each other?

Valid Certificate

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

INDEPENDENT-SET

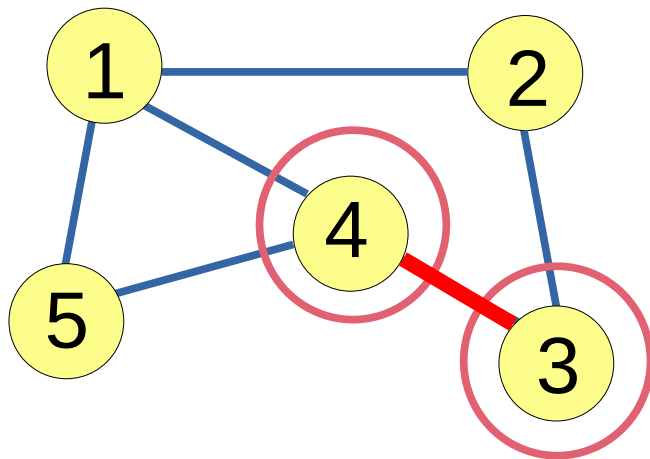Exists a set of k vertices s.t. no two are neighbors of each other?

Invalid Certificate

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

Independent-Set Instance and Certificate

Instance: graph G and natural k

Independent-Set Instance and Certificate

Instance: graph **G** and natural **k**

Certificate: subset of vertices of **G**

Independent-Set Instance and Certificate

Instance: graph G and natural k

Certificate: subset of vertices of G

Assertion for valid certificate C of (G,k):
Forall e in edges of G:

INDEPENDENT-SET Instance and Certificate

Instance: graph G and natural k

Certificate: subset of vertices of G

Assertion for valid certificate C of (G,k):
  Forall e in edges of G:
    Not (And <u>one vertex of</u> e in C
           <u>the other vertex of</u> e in C)

Independent-Set Instance and Certificate

Instance: graph G and natural k

Certificate: subset of vertices of G

Assertion for valid certificate C of (G,k):
   Forall e in edges of G:
      Not (And <u>one vertex of</u> e in C
                <u>the other vertex of</u> e in C)
and
   Size of C >= k

3-SAT – **Mother of All NP-Problems**

Exists true/false assignment of the variable satisfying all clauses?

$$(\neg x_1 \vee x_2 \vee x_3)$$

$$(x_1 \vee \neg x_2 \vee x_4)$$

$$(x_2 \vee \neg x_3 \vee \neg x_4)$$

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

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$$(x_2 \lor \neg x_3 \lor \neg x_4)$$

Valid or Not?

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

$$x_1 \rightsquigarrow F \qquad x_2 \rightsquigarrow T \qquad x_3 \rightsquigarrow F \qquad x_4 \rightsquigarrow F$$

Certificate: Assignment from variables of the CNF to Boolean

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Certificate: Assignment from variables of the CNF to Boolean

3-Sat – **Mother of All NP-Problems**

Exists true/false assignment of the variable satisfying all clauses?

$$(\neg x_1 \lor x_2 \lor x_3) \checkmark$$

$$(x_1 \lor \neg x_2 \lor x_4)$$

$$(x_2 \lor \neg x_3 \lor \neg x_4)$$

Valid or Not?

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

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Certificate: Assignment from variables of the CNF to Boolean

3-SAT – Mother of All NP-Problems

Exists true/false assignment of the variable satisfying all clauses?

$$(\neg x_1 \lor x_2 \lor x_3) \;✔$$

$$(x_1 \lor \neg x_2 \lor x_4)$$

$$(x_2 \lor \neg x_3 \lor \neg x_4) ✔$$

Valid or Not?

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Valid or Not?

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

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$$(x_1 \vee \neg x_2 \vee x_4)$$

$$(x_2 \vee \neg x_3 \vee \neg x_4) \checkmark$$

Invalid Certificate

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

$$x_1 \rightsquigarrow \text{F} \qquad x_2 \rightsquigarrow \text{T} \qquad x_3 \rightsquigarrow \text{F} \qquad x_4 \rightsquigarrow \text{F}$$

Certificate: Assignment from variables of the CNF to Boolean

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$$(\neg x_1 \vee x_2 \vee x_3)$$

$$(x_1 \vee \neg x_2 \vee x_4)$$

Valid or Not?

$$(x_2 \vee \neg x_3 \vee \neg x_4)$$

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

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Certificate: Assignment from variables of the CNF to Boolean

3-SAT – Mother of All NP-Problems

Exists true/false assignment of the variable satisfying all clauses?

$$(\neg x_1 \lor x_2 \lor x_3) \checkmark$$

$$(x_1 \lor \neg x_2 \lor x_4)$$

$$(x_2 \lor \neg x_3 \lor \neg x_4)$$

Valid or Not?

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$$x_1 \rightsquigarrow \text{F} \qquad x_2 \rightsquigarrow \text{F} \qquad x_3 \rightsquigarrow \text{F} \qquad x_4 \rightsquigarrow \text{F}$$

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Valid or Not?

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$$(x_1 \lor \neg x_2 \lor x_4) \checkmark$$

$$(x_2 \lor \neg x_3 \lor \neg x_4) \checkmark$$

Valid Certificate

Instance: A Boolean formula in 3-conjunctive normal form (CNF)

$$x_1 \rightsquigarrow \text{F} \qquad x_2 \rightsquigarrow \text{F} \qquad x_3 \rightsquigarrow \text{F} \qquad x_4 \rightsquigarrow \text{F}$$

Certificate: Assignment from variables of the CNF to Boolean

3-Sat Instance and Certificate

Instance: 3CNF formula Phi

3-Sat Instance and Certificate

Instance: **3CNF formula** **Phi**

Certificate: **mapping from variables of Phi to Booleans**

3-SAT Instance and Certificate

Instance: 3CNF formula Phi

Certificate: mapping from variables of Phi to Booleans

Assertion for valid certificate C of Phi:
Forall c in clauses of Phi:
Exists (literal l in c s.t
l is satisfied under c)

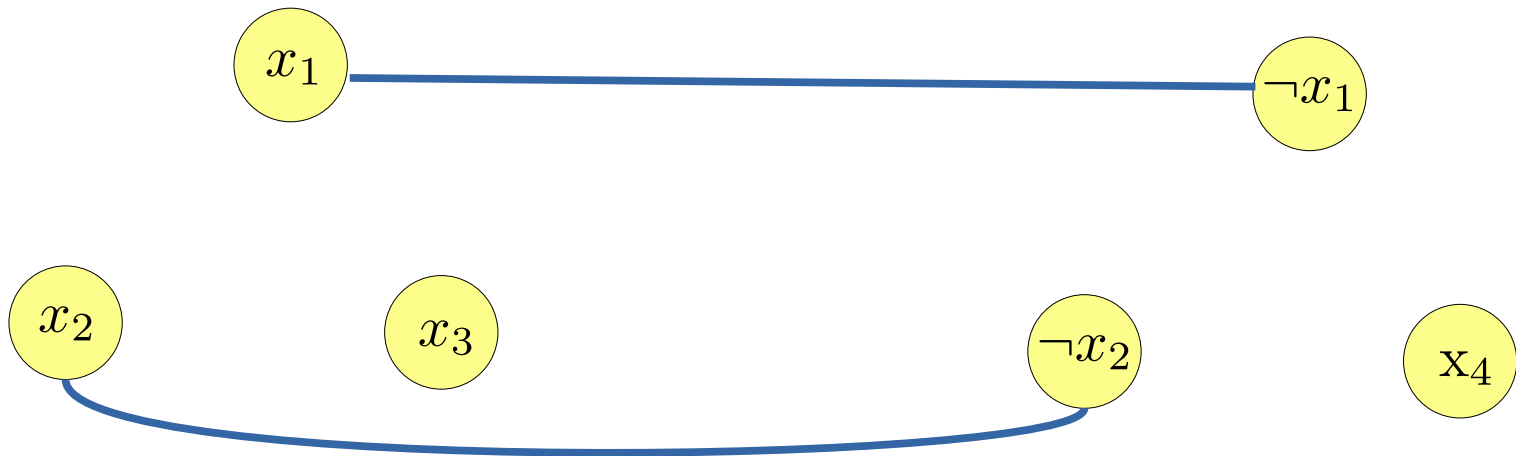# Reductions

Instance construction
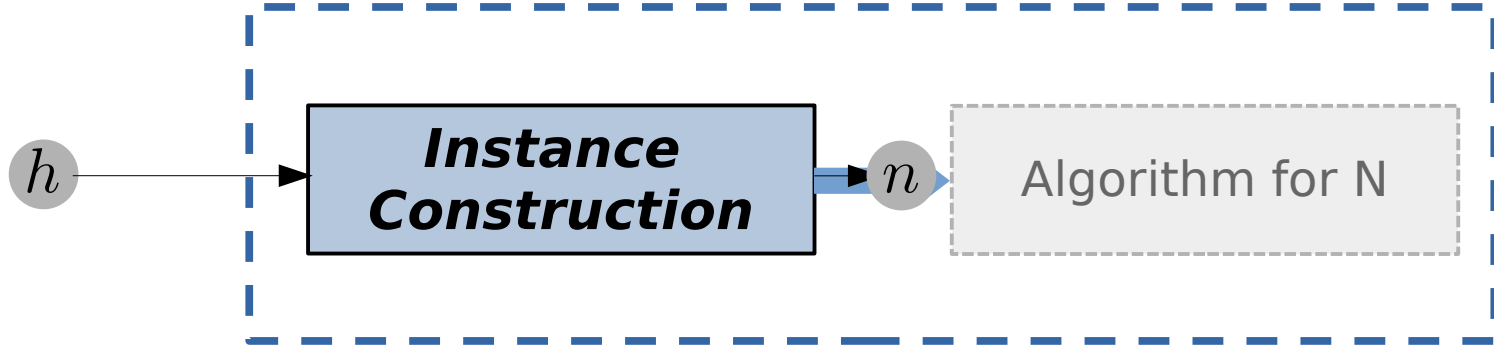
Justifying N No => H No
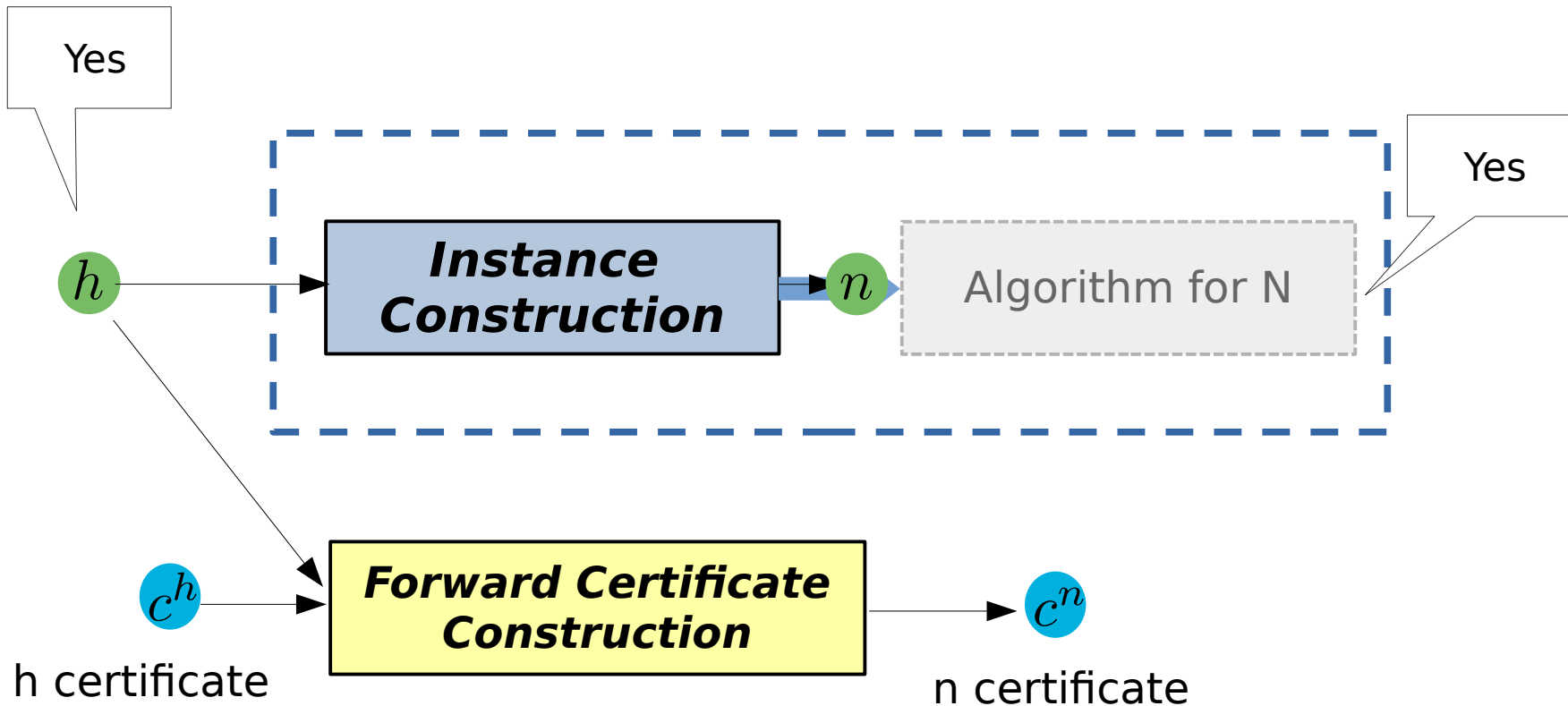
Justifying N Yes => H Yes
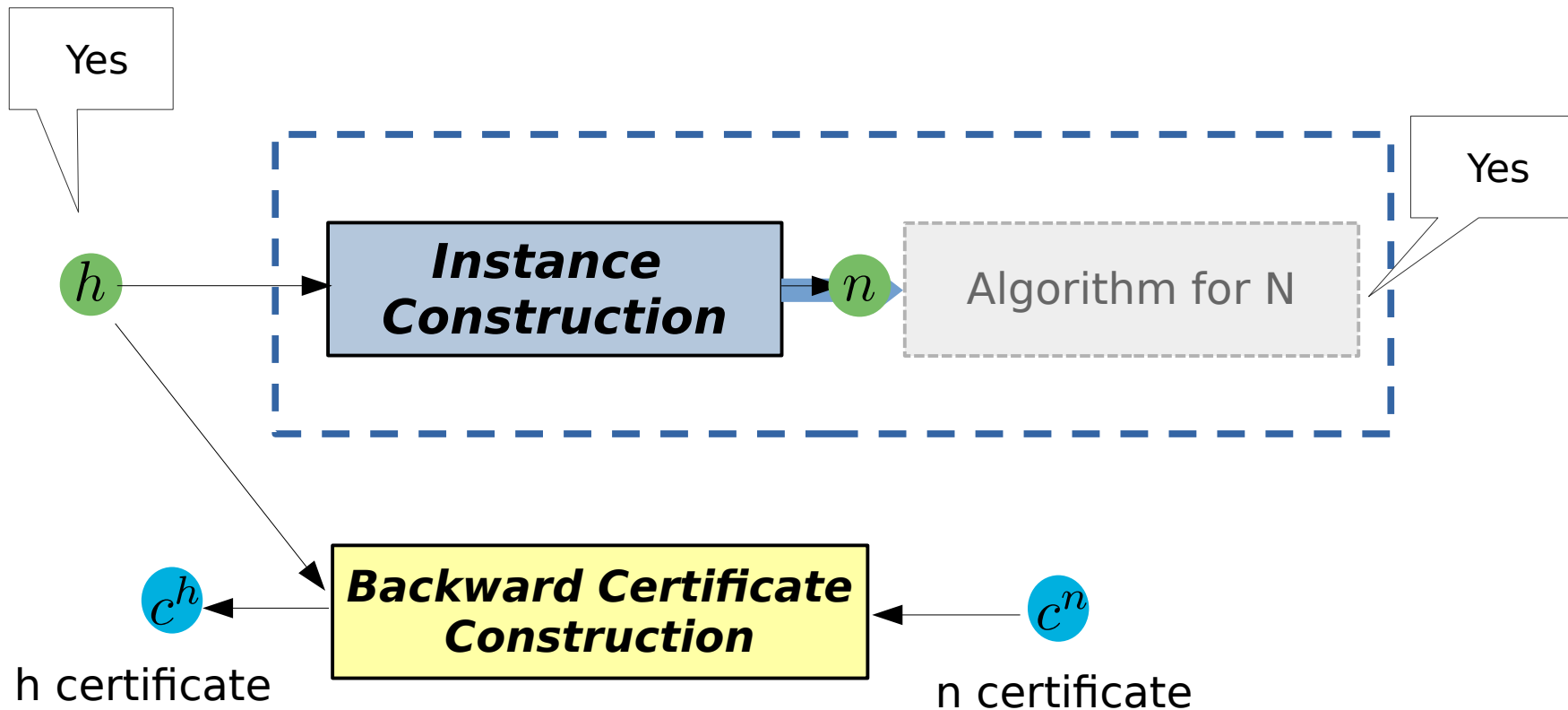
Instance Construction
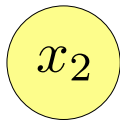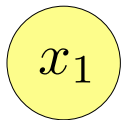
$$(x_1 \vee x_2 \vee x_3)$$

$$(\neg x_1 \vee \neg x_2 \vee x_4)$$
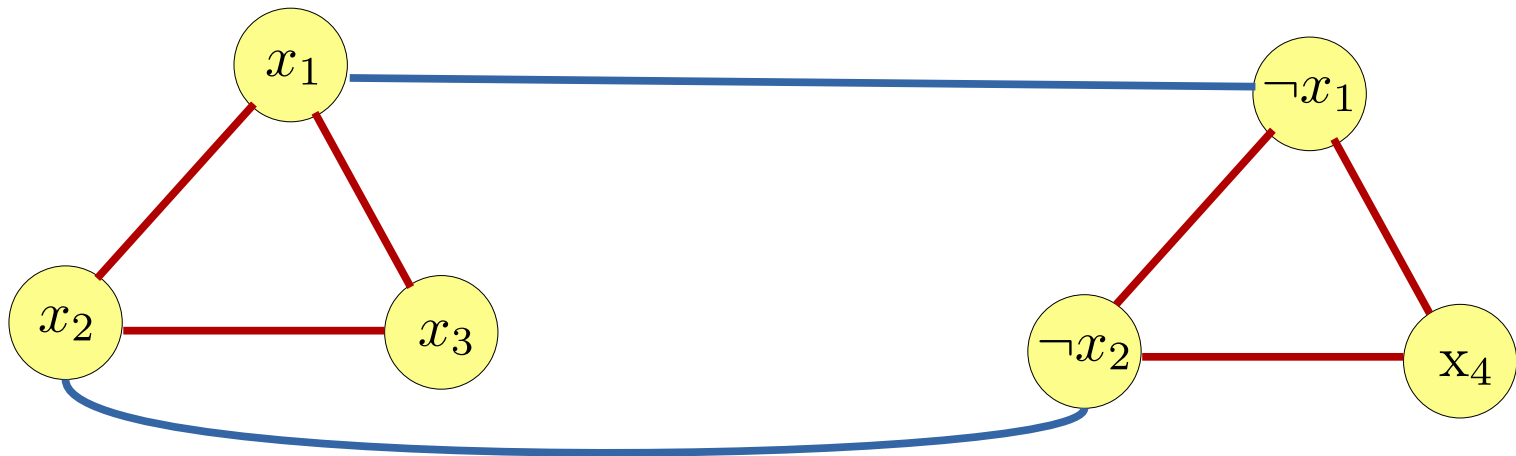
Instance Construction

$$(x_1 \lor x_2 \lor x_3)$$

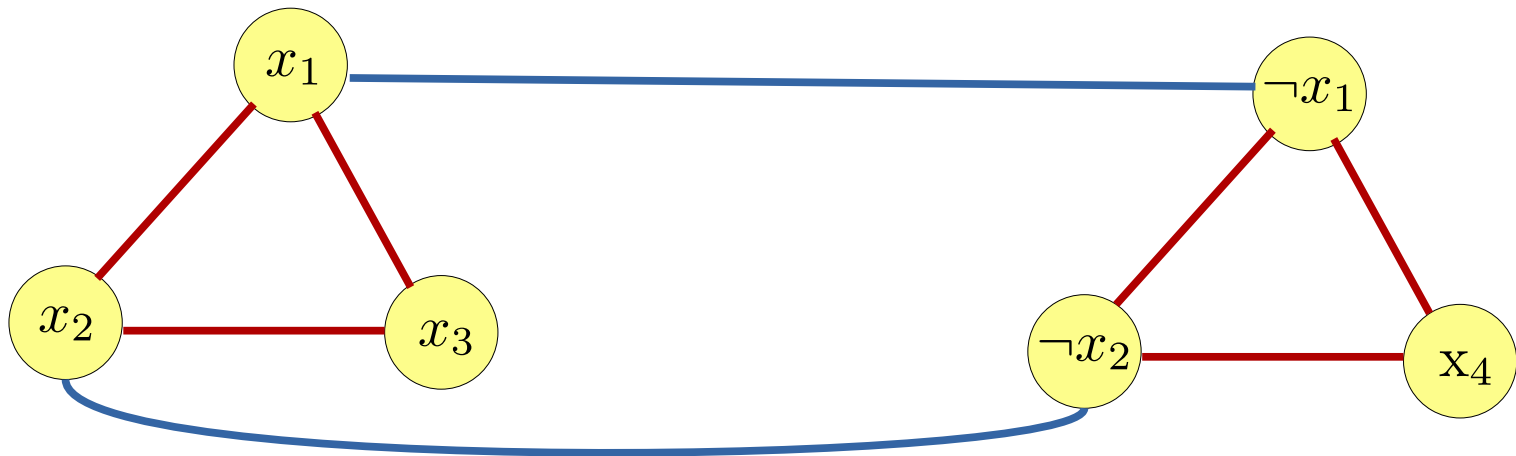$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

x_1

x_2

x_3

Instance Construction

$$(x_1 \lor x_2 \lor x_3)$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

x_1

$\neg x_1$

x_2

x_3

$\neg x_2$

x_4
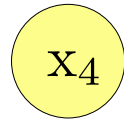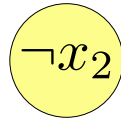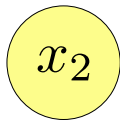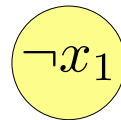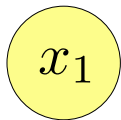
Instance Construction

$$(x_1 \lor x_2 \lor x_3)$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

Instance Construction

$$(x_1 \lor x_2 \lor x_3)$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

Forward Certificate Construction

$$(x_1 \lor x_2 \lor x_3)$$

$$x_1 = T$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

$$x_4 = T$$

Forward Certificate Construction
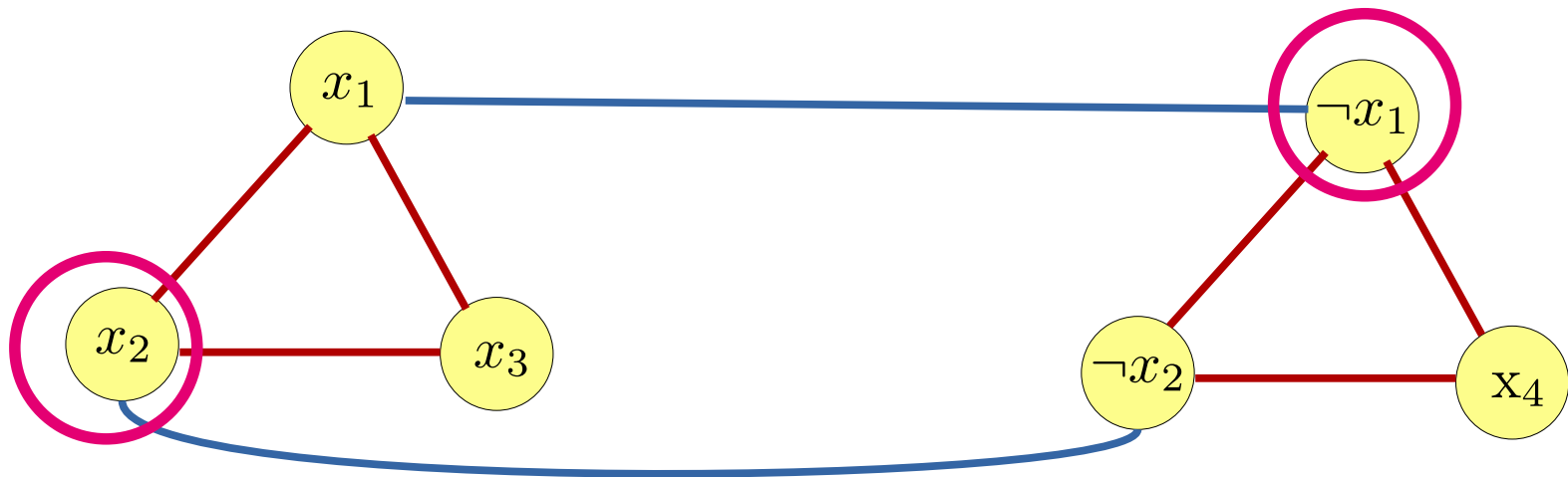
$$(x_1 \lor x_2 \lor x_3) \qquad x_1 = T$$
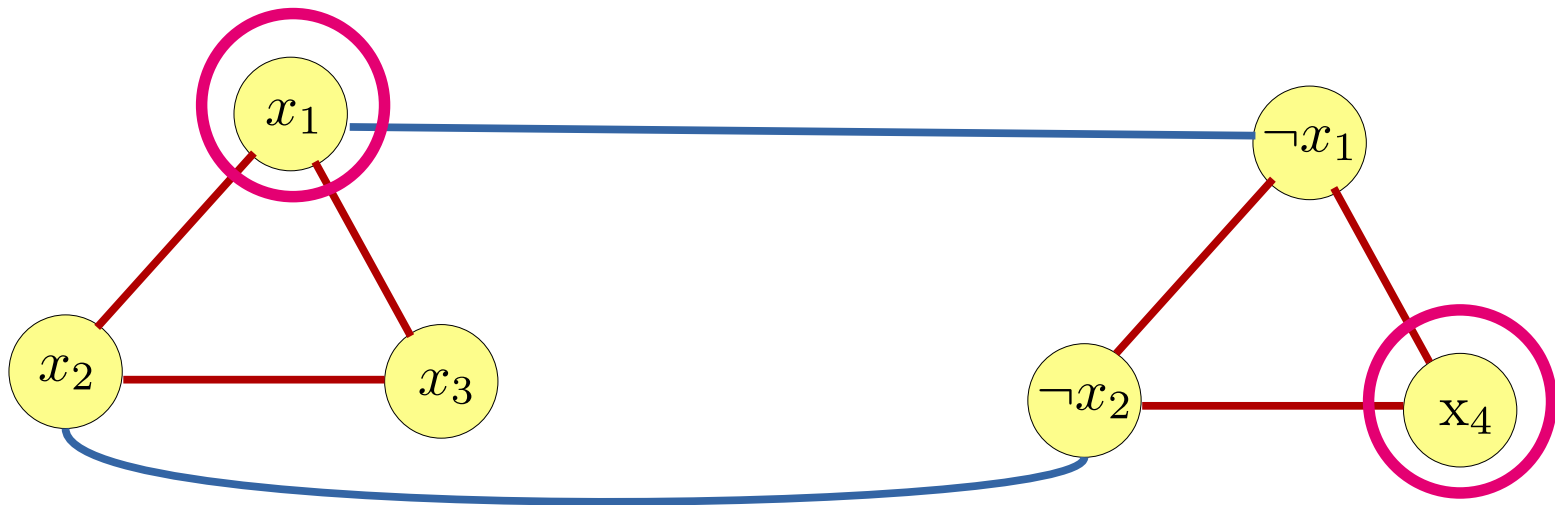
$$(\neg x_1 \lor \neg x_2 \lor x_4) \qquad x_4 = T$$

Backward Certificate Construction
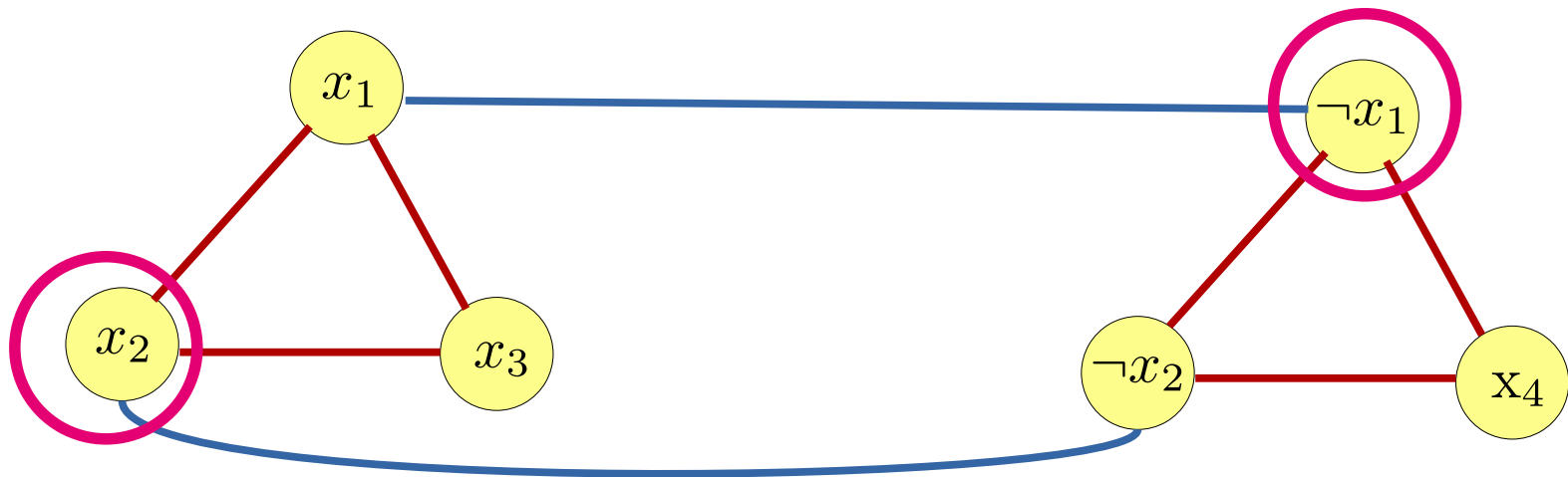
$$(x_1 \lor x_2 \lor x_3)$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

Backward Certificate Construction

$$(x_1 \vee x_2 \vee x_3) \qquad x_2 = T$$

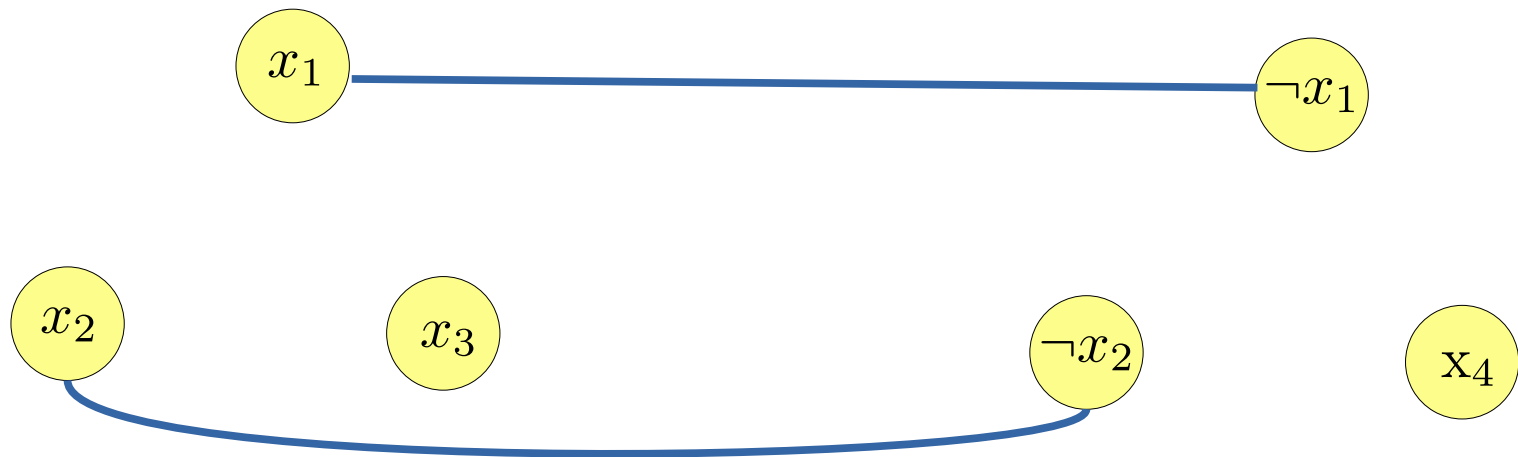$$(\neg x_1 \vee \neg x_2 \vee x_4) \qquad x_1 = F$$

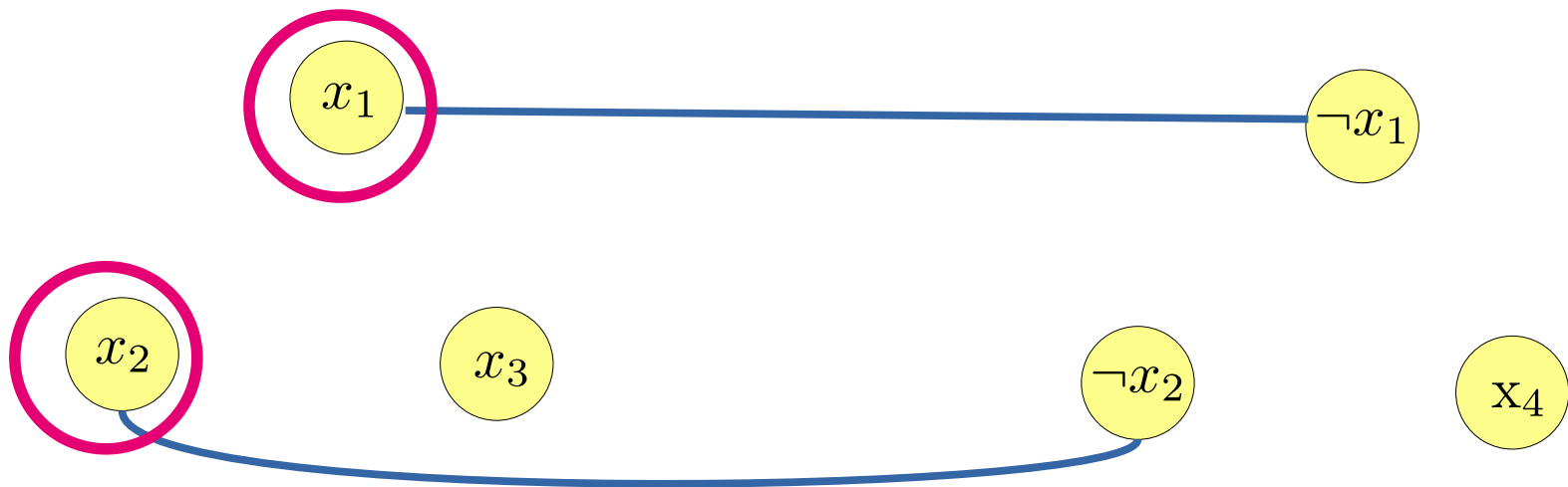# Does this also works?

$$(x_1 \lor x_2 \lor x_3)$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

Does this also works?

$$(x_1 \vee x_2 \vee x_3)$$
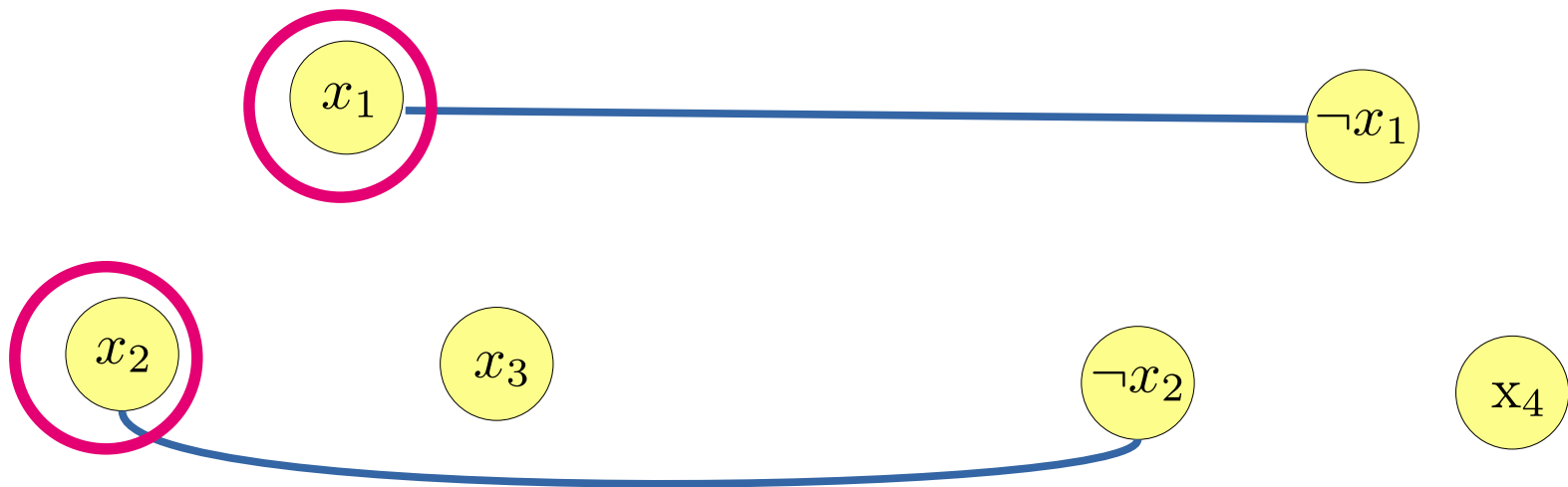
$$(\neg x_1 \vee \neg x_2 \vee x_4)$$

Does this also works?

$$(x_1 \lor x_2 \lor x_3) \qquad x_1 = T \quad x_2 = T$$

$$(\neg x_1 \lor \neg x_2 \lor x_4) \qquad x_3 = x_4 = F$$

Does this also works?

$$(x_1 \lor x_2 \lor x_3)$$

$$x_1 = T \quad x_2 = T$$

$$(\neg x_1 \lor \neg x_2 \lor x_4)$$

$$x_3 = x_4 = F$$