15 August 1997

ELSEVIER

Optics Communications 141 (1997) 17–20

OPTICS COMMUNICATIONS

Numerical analysis of vectorial wave propagation in waveguides with arbitrary refractive index profiles

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Received 31 July 1996; revised 17 April 1997; accepted 21 April 1997

Abstract

We demonstrate an efficient numerical method for calculating transverse electric (TE) or transverse magnetic (TM) modal properties in a waveguide with an arbitrary refractive index distribution. We approximate a planar waveguide cross-section by a finite number of thin dielectric layers and use 2 × 2 transfer matrices to relate adjacent layers. Using a back-propagation method that is simple and computationally efficient, we solve for the vectorial field across the multi-layered waveguide and plot the mode profile. There is rapid convergence to a high-accuracy solution. We also obtain the number of guided modes and the propagation constant of each mode. © 1997 Elsevier Science B.V.

1. Introduction

We present a numerical guided-wave calculation based on the transfer matrix method [1] to calculate the mode profile in a planar waveguide with a varying refractive index. We approximate the waveguide by a finite series of dielectric layers, each characterized by a discrete refractive index and width. We place the multi-layered waveguide structure between two high-index regions as shown in Fig. 1. The input field is introduced from the high-index region via evanescent mode coupling, and the incidence angle θ_{in} is varied to excite all possible modes in the waveguide. The waveguiding layers and the high-index regions are separated by a low-index cladding gap. This configuration is directly analogous to prism coupling to a waveguiding layer using a high-index prism [2]. Using the incident angle value corresponding to the desired mode, we find the spatial field profile of the propagating mode.

Three useful parameters in a multi-layered waveguide structure are the number of guided modes (if any), the propagation constant of a mode, and the mode width for TE or TM polarizations. Analytical methods using the matrix approach solve the characteristic equation for the layers and find the propagation constants from the transfer matrix zeros [3,4]. This approach can become complicated as the number of layers increase since it involves the solution of complicated differential or transcendental equations. Numerical matrix methods can solve for a particular propagation constant or mode profile [5,6] but may not be able to determine all guided modes. The algorithm presented here can be written as a short program (e.g., a 150-line Mathematica program) executed on a personal computer and can yield the number of guided modes, the propagation angle for each guided mode, the propagation constant, and the mode profile within several minutes for 100 layers. High-accuracy solutions are obtained for vectorial wave propagation in the multi-layered structures. The waveguide structure can be complicated, asymmetric and have an arbitrary index variation that is continuous or abrupt. The structure can be either a strongly-guided waveguide or a weakly-guided (rib) waveguide. Some examples of complicated waveguide structures are multi-layered thin-films with different refractive indices, curved waveguides that have an exponentially varying equivalent index profile (such as ring lasers or S-curve connections in optical circuits) [7], or waveguiding layers with loss or gain parameters. Even though this method is a one-dimensional waveguide calculation, it is easily extended to two

PII S0030-4018(97)00208-3

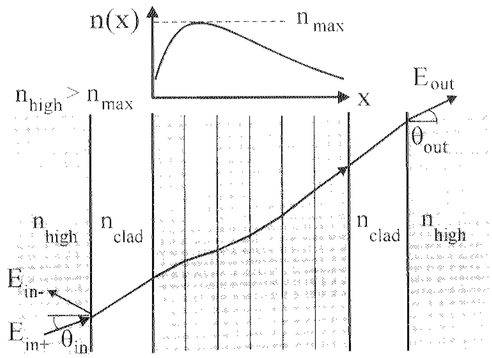

Fig. 1. A waveguide cross-section with an arbitrary index of refraction profile is divided into several layers. The input field E_{in} and output field E_{out} are coupled in and out of the waveguide from the high-index regions via the adjacent cladding layers.

dimensions by using the effective index method [8] to incorporate the second dimension.

2. Transfer matrix formulation

The waveguiding region has an arbitrarily varying index of refraction and is modeled as many thin dielectric layers, each layer with a discrete refractive index. Each layer has a complex transfer matrix that relates the transmitted and reflected fields within that layer to the fields in the adjacent layer. The transfer matrix method allows us to find the cumulative field amplitude at any point in the waveguide by multiplying the transfer matrices.

The numerical method presented here assigns a transmitted field at the output and propagates backwards through the waveguide layers to find the total field at the input. In Fig. 1, we see the case of a waveguide of arbitrary index distribution $n(x)$ separated from a high-index region by a thin cladding layer (such as an air gap) on either side. The field is coupled in and out of the waveguide via the cladding layers with index n_{clad} on either side. To excite all possible guided modes, the refractive index of the input layer, n_{high} must be greater than the maximum refractive index in the waveguide region, n_{max}. By assigning a transmitted electric field amplitude E_{out} and propagating backwards, we obtain both the transmitted and reflected waves at the input of the waveguide, E_{in+} and E_{in-}.

We calculate the ratio of the output field to the total input field at the final air gap as a function of incidence angle θ_{in}. The angle for the lowest-order guided mode corresponds to the largest angle that results in a local maximum for the field excitation in the waveguide (and hence a local maximum transmittance in the waveguide). We then use this angle to propagate through the medium and calculate the field at each layer in the waveguide as a function of space.

Suppose there are two layers in the waveguide as shown in Fig. 2. The first layer has an index of n_g and the second has a length d and index n_h. With an incidence angle θ_i, the transmission angle into the next layer is given by $\theta_t = \arcsin[(n_g \sin\theta_i)/n_h]$. The TE propagation vectors on each side of the interface are $g = (2\pi n_g \cos\theta_i)/\lambda$ and $h = (2\pi n_h \cos\theta_t)/\lambda$. The TM propagation vectors are $g = (2\pi\cos\theta_i)/\lambda n_g$ and $h = (2\pi\cos\theta_i)/\lambda n_h$. The transmission and reflection coefficients at the interface are $T = 2g/(g+h)$ and $R = (g-h)/(g+h)$ for the left-hand side of the interface and $T_p = 2h/(g+h)$ and $R_p = (h-g)/(g+h)$ for the right-hand side of the interface. The total field amplitude on the left side of the interface is composed of two elements denoted by E_i^+ and E_i^-. The total electric field amplitude on the right side of the interface is composed of E_{i+1}^+ and E_{i+1}^-. The equations relating the waves across the interface are $E_i^- = RE_i^+ + T_p E_{i+1}^-$ and $E_{i+1}^+ = R_p E_{i+1}^- + TE_i^+$.

Using these two equations, the transfer matrix at this interface is:

$$A = \begin{bmatrix} \exp(jhd) & 0 \\ 0 & \exp(-jhd) \end{bmatrix} \cdot \begin{bmatrix} \dfrac{TT_p - RR_p}{T_p} & \dfrac{R_p}{T_p} \\ -\dfrac{R}{T_p} & \dfrac{1}{T_p} \end{bmatrix}, \tag{1}$$

with the fields in the adjacent layer given by the relation

$$\begin{bmatrix} E_{i+1}^+ \\ E_{i+1}^- \end{bmatrix} = A \cdot \begin{bmatrix} E_i^+ \\ E_{i-} \end{bmatrix}. \tag{2}$$

The matrix for each segment has similar coefficients depending on the length and refractive index of each segment. We can also include any parameters for gain or loss in the layers in the transfer matrices. The transmission angle θ_t for one interface becomes the input angle θ_i for

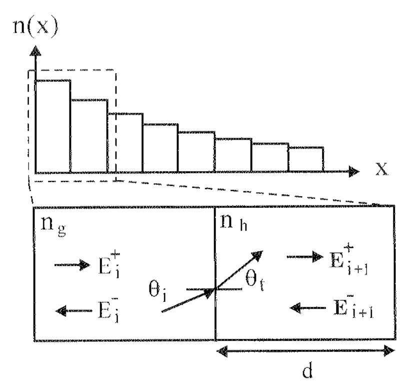

Fig. 2. An arbitrary index distribution is divided in a finite number of layers with a discrete refractive index value assigned to each section. The inset shows the interface between two adjacent layers with indices n_g and n_h and the forward and backward-propagating fields in each layer.

the next interface. Once we have computed a transfer matrix for each segment, the final output for $N-1$ layers is

$$\begin{pmatrix} E_N^+ \\ E_N^- \end{pmatrix} = A_{N-1} \cdot A_{N-2} \cdot \ldots A_1 \cdot \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix} = A_{\text{total}} \cdot \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix}. \tag{3}$$

By storing the field amplitude value at each interface, we can plot the field as a function of space in order to observe the mode profile.

3. Method outline and example

The calculation steps are as follows and the variables and fields are indicated in Fig. 1. Although the method is applicable to either TE or TM modes, we perform the numerical calculations in the following examples for the TE guided wave with the free-space wavelength $\lambda_0 = 1.55$ μm. We describe the total field at each point in space by $E(x) = E^-(x) + E^+(x)$ and denote it by $[E_-(x), E_+(x)]$.

3.1. Possible excited modes in a multilayer waveguide

The two input parameters required are the structure of the waveguide layers and the incident guiding angle. The structure is a list, $L = \{\{n_{in}, l_{in}\}, \{n_{clad}, l_{clad}\}, \{n_1, l_1\}, \{n_2, l_2\}, \ldots, \{n_{N-1}, l_{N-1}\}, \{n_{clad}, l_{clad}\}, \{n_{out}, l_{out}\}\}$, that describes the refractive index and thickness of each layer. We choose an initial θ_{in} and using Snell's law, we propagate through L starting at the end of the structure list until we solve for θ_{out}. n_{in} and n_{out} are the refractive indices of the outer high-index layers.

We assign a wave $E_{out} = [E_{out-}, E_{out+}] = [0,1]$ and propagate backwards through the layers to solve for the field $E_{in} = [E_{in-}, E_{in+}]$ using θ_{out} as the starting angle. E_{in+} is the wave entering the waveguiding layers. We store the value of $|E_{out+}/E_{in+}|^2$ for θ_{in}, increment θ_{in}, and repeat the calculation. This result is the transmittance through the structure for varying θ_{in}. The number of transmittance peaks for the range of possible incident angles indicates the total number of supported modes in the waveguide. The incidence angle for the lowest-order mode is given by the largest angle at which there is a peak in the field transmittance output.

The lowest-order modes are not as lossy as the higher-order modes and appear as much smaller peaks. Therefore, to identify the lowest-order mode angle, we decrease the air gap so that we increase the output coupling at the air gap and identify the small peaks. Once we identify the approximate value of the guiding angle, we increase the air gap to find a more accurate solution. As the air gap is increased, the peaks narrow and the peak values slightly shift (typically less than one percent for an increase of the gap from 0.05 to 0.5 μm). The guided mode becomes increasingly sensitive to the guiding angle and the peaks narrow because we are decreasing the output coupling

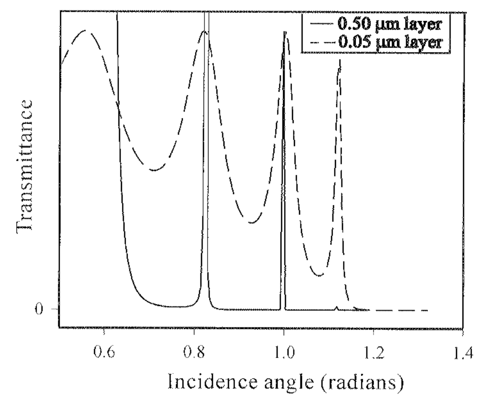

Fig. 3. The field transmittance is graphed as a function of the incidence angle for two different cladding layer (air gap) thicknesses. The peaks indicate field excitation and correspond to guided modes. The small peak at the largest angle shown indicates guiding for single-mode propagation. The plots are normalized to the second peak from the right.

from the waveguide and effectively increasing the photon lifetime in the waveguiding region. Increasing the gap size narrows the peaks and rapidly converges to a high-accuracy solution. In Fig. 3, we see the field intensity output as a function of incident angle for a simple 1-μm-wide planar waveguide of refractive index $n = 3.3$. The cladding layers in this example are air, but they can be any low-index material. The output is shown for an air gap of 0.05 μm and an air gap of 0.5 μm, and the peaks correspond to guided modes. All calculations are for a free-space wavelength of $\lambda_0 = 1.55$ μm.

3.2. Field mode profile calculation

By propagating backwards up to any particular layer, we find a value for the total field and the propagation angle at that layer. We call this field $E_{wg} = [E_{wg-}, E_{wg+}]$. The transmittance at this point is $|(E_{wg+} + E_{wg-})/E_{in+}|^2$. Using the guiding angle for the lowest order mode, we solve for $[E_{wg-}, E_{wg+}]$ after every layer in the structure list L. We plot $|E_{wg+} + E_{wg-}|^2$ versus the length after each increment to get the mode profile. Note that since we know the refractive index of each layer and can solve for the propagation angle in each layer for a particular mode, we can also solve for the propagation constant of that mode $k = (w/c) n_{layer} \sin\theta_{layer}$. This propagation constant is matched throughout all of the layers. At the air gap, the propagation angle is a complex value $\theta = \theta_R + i\theta_I$ indicating that the field at that point is a decaying field.

4. Numerical examples

4.1. Step-index waveguide

In Fig. 4, we calculate the mode profile for a 1-μm-wide planar waveguide of effective index $n_1 = 3.3$ with $\lambda_0 =$

1.55 μm. The cladding layers are air ($n = 1.0$). We compare the mode profile with the exact solution obtained by solving the wave equations inside and outside the physical waveguide. We solve the numerical method for a step-index waveguide surrounded by air gaps of widths 0.05, 0.1, and 0.5 μm. The corresponding incident angles for the three cases are $\theta_{in} = 1.117049$, 1.115379, and 1.114346 rad, respectively. The propagation angle inside the waveguiding region is $\theta_1 = 1.36660$ (for the latter θ_{in}). The propagation constant for this mode in the waveguide is $k = (w/c)n_1 \sin\theta_1$. As the air gap is increased, the accuracy of the incident angle increases indefinitely. The case with the 0.5-μm air gap agrees with the exact solution and the error in the propagation angle is on the order of 10^{-6}. The error is due to the finite incident angle increments in numerically finding the incident angle, and the accuracy can be further increased by using higher resolution increments.

4.2. Quadratic-index waveguide

The quadratic-index distribution is $n(x) = n(1 - n_2 x^2/n)^{1/2}$. The solution of the wave equation $\partial^2 E/\partial x^2 + [k^2(1 - (n_2/n)x^2)]E = 0$ is of the form $E = \exp(-x^2/\omega^2)$ where $\omega^2 = (\lambda/\pi)\sqrt{nn_2}$. The chosen index distribution is $n(x) = 3.4(1 - x^2/3.4)^{1/2}$ as shown in the inset of Fig. 5. The numerical transfer matrix method is applied by dividing the quadratic-index distribution into 0.01-μm-thick step-index layers. The waveguide formed by the index distribution has a 0.1-μm air gap on either side. The incident angle for single-mode propagation is $\theta_{in} = 1.16226$. From this angle, we can find the angle in any layer in the waveguide and find the real or complex propagation constant for that layer. If the propagation constant is real, the mode is guided and the propagation constants in the guiding layers are matched and given by $k = (w/c)n_{layer} \sin\theta_{layer}$. In Fig. 5, we see that the intensity mode profiles found by the numerical matrix calculation and the exact solution are in agreement.

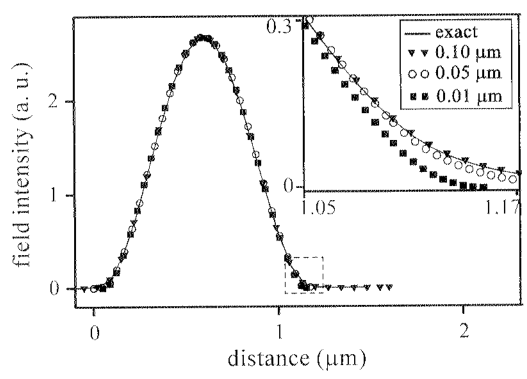

Fig. 4. The field intensity is graphed as a function of space showing the mode profile for a step-index planar waveguide. The solid line is the known exact solution and the symbols represent the numerical transfer matrix method solution for different cladding-layer thicknesses as indicated on the graph. The inset shows an enlargement of the curves at the base of the profile.

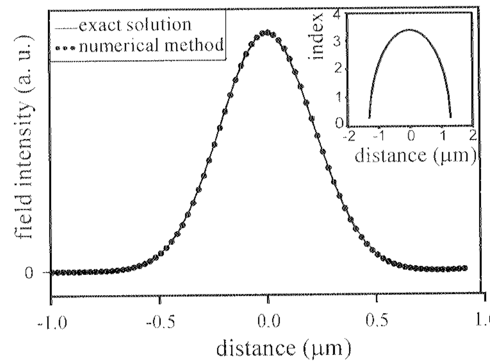

Fig. 5. The field intensity is graphed as a function of space showing the mode profile of a waveguide with a quadratic index of refraction distribution. The solid line is the exact solution and the solid circles represent the numerical transfer matrix method solution. The inset shows the quadratic refractive index distribution as a function of space across the waveguide.

5. Conclusion

The numerical method presented here is an accurate and fully vectorial wave calculation using 2 × 2 transfer matrices. It is a simple, fast, and general method to analyze multi-layered waveguides with any refractive index distribution. By assigning an output field and back-propagating through the waveguide layers, we calculate the incident angles for all possible propagating modes, the mode profile of any mode, and the propagation constant of the mode. Intensity mode profiles of quadratic-index and step-index examples calculated using this method agree with the exact solutions.

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