Scheme for realizing a photon number amplifier

Seng-Tiong Ho and Horace P. Yuen

Department of Electrical Engineering and Computer Science, The Robert R. McCormick School of Engineering and Applied Science, The Technological Institute, Northwestern University, Evanston, Illinois 60208

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A scheme for realizing a photon number amplifier by use of a high-quantum-efficiency photodetector and a number-state semiconductor laser is analyzed. It is found that the photon number amplifier is not significantly limited by the electronic amplifier noise or by the laser quantum efficiency for input states that are nearly classical.

It was suggested by Yuen[1-3] that a photon number amplifier (PNA) that transforms an input state $|n\rangle$ with n photons to an output state $|Gn\rangle$ with Gn photons, where $G > 1$ is the gain factor, is an ideal amplifier for direct detection of any signal, whether it is classical or squeezed or whether it is phase coherent or incoherent.[2,3] It is an ideal amplifier in that the signal-to-noise ratio (SNR) will not be degraded after the amplification. It is unlike the usual optical power amplifier, in which a 3-dB degradation in the SNR is unavoidable. However, until now to our knowledge no such PNA has been realized. In this Letter we show that one can realize a PNA by using a high-quantum-efficiency photodetector to detect the incoming photons. The photocurrent is then amplified by use of a current amplifier. Finally the amplified current is used to drive a number-state semiconductor laser to generate the amplified output state. The number-state laser was realized by Yamamoto *et al.*,[4] and thus the scheme suggested here should be possible in practice. We also analyze the fidelity of the number-state amplifier as a function of the photodetection quantum efficiency, electronic amplifier noise, number-state laser efficiency, and the quantum nature of the input state. It is found that the PNA is not significantly limited by the electronic amplifier noise and the laser quantum efficiency for input states that are classical or nearly classical.

The PNA realization we propose to study is depicted in Fig. 1. An input field is detected with a photodetector with quantum efficiency ξ, and the output current is electronically amplified with a power gain G. A laser such as the near-number-state diode laser studied by Yamamoto *et al.*[4] that generates the output photon number in scaled proportion to the driven current is used to yield the output field. Ideally, if a perfect photodetector ($\xi = 1$) converts an input photon number n to a current I, then a driven current GI would generate an output field with Gn photons from a number-state laser. Thus Fig. 1 can provide a perfect PNA realization in principle, with its rate limitation determined by the photodetector bandwidth and the laser response time. In practice only a quantum efficiency $\xi \sim 90\%$ can be obtained at present even for photodetectors with no gain. Furthermore the sub-Poissonian photon number fluctuation reduction that has been achieved thus far in an intensity-squeezed diode laser is less than 10 dB.[5] Thus the PNA that can be built this way is limited in its output noise performance. In the following we will calculate the effects of nonunity photodetector quantum efficiency ξ and nonideal number-state lasers on the amplifier noise performance. But first we need a good representative of a nonideal number-state laser.

There are two types of loss in a number-state diode laser. In the first type, some electrons in the driven current miss the lasing region of the p-n junction in the diode; thus no photon would be generated for sure. In the second type, the electrons hit the proper lasing region and would generate photons with a quantum efficiency, which is just the reverse situation of photodetection. This situation is depicted in Fig. 2 and schematically represented in Fig. 3. In a perfect electron–photon or electrical–optical conversion, the current I, optical power P, and total photon number N are related by

$$I = eN/t\,, \tag{1}$$

$$P = \hbar\omega I/c\,, \tag{2}$$

where e is the electronic charge, ω is the optical frequency and t is the time duration under consideration. In Fig. 3 the input current I can be represented by a photon-annihilation operator c with $N = c^{\dagger}c$ by means of Eq. (1). The output photon-annihilation operator d can be related to c through an intermediate photon-annihilation operation c' as follows. The second type of loss is similar to that of photodetection; thus the effect of η_2 is equivalent to random detection of the electron with probability $1 - \eta_2$. As for the photodetector case,[6] we can write

$$d = {\eta_2}^{1/2}c' + (1 - \eta_2)^{1/2}v\,, \tag{3}$$

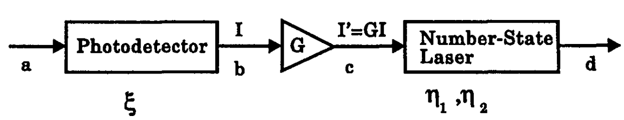

Fig. 1. PNA realization. The input signal mode a is photodetected with quantum efficiency $\xi = 1$, and the output current I is electrically amplified with a power gain G, which then drives a number-state laser characterized by two quantum efficiencies η_1 and η_2, yielding the amplified output mode d.

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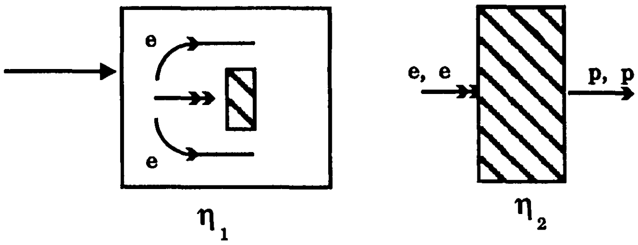

Fig. 2. Two types of loss in a number-state diode laser. In the first type the electron e misses the lasing junction with probability $1 - \eta_1$. In the second type a photon p is generated with probability η_2 when an electron hits the lasing junction.

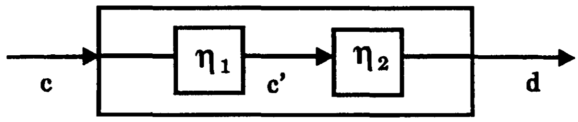

Fig. 3. Two different types of loss represented sequentially with an intermediate mode c'.

where v is the photon-annihilation operator of an independent vacuum mode. For the first type of sure detection, we write

$$c' \simeq \eta_1^{1/2} c\,, \tag{4}$$

which implies that

$$N_c' \equiv c'^{\dagger} c' \simeq \eta_1 N_c\,. \tag{5}$$

Note that relation (4) or (5) cannot be exactly correct as a quantum operator because the commutation rule $[c', c'^{\dagger}] = I$ is violated in relation (4) and the spectrum of $N_{c'}$ is no longer the set of integers, assuming N_c has the integer spectrum as a number operator in relation (4). However, when they are combined with Eq. (3), we obtain

$$d = (\eta_1 \eta_2)^{1/2} c + (1 - \eta_2)^{1/2} v\,, \tag{6}$$

which expresses the situation we wish to represent: loss through η_1 does not introduce any output photon number fluctuation, whereas loss through η_2 does. From this interpretation, Eq. (6) should give the correct result, at least for computing the output photon number $N_d = d^{\dagger} d$ fluctuation independently of the operator validity of relations (4) and (5).

Next we model the electronic amplifier with gain G. The input field mode to the photodetector with quantum efficiency ξ has a photon-annihilation operator a, with output photocurrent I represented by a photon-annihilation operator b. The electronic amplifier with gain G is represented as

$$N_c = G N_b + n\,, \tag{7}$$

where N_c is the number operator corresponding to the output current I' and n is the additive noise introduced by the electronic amplifier. Again note that Eq. (7) is not a generally valid quantum-mechanical representation, as the spectra of N_c and N_b cannot both be the set of integers. However, Eq. (7) can be used at least for the purpose of computing photon number fluctuations. Generally, if there is any doubt concerning the validity of Eqs. (6) and (7) as quantum operator equations, we can always reinterpret them as classical equations for the field amplitudes or photon numbers. The conclusions drawn from these equations are then valid in the case of photon number quantum measurements, which are really the only processing for which one would employ a PNA rather than another kind of amplifier.[6]

The output photon number fluctuation can now be readily computed from Eqs. (6) and (7), with the photodetector input and output related as usual by

$$b = \xi^{1/2} a + (1 - \xi)^{1/2} u\,, \tag{8}$$

and an independent vacuum mode with photon annihilation operator u. From Eq. (8) we obtain

$$\begin{aligned} N_b = {} & \xi N_a + (1 - \xi) N_u \\ & + \xi^{1/2} (1 - \xi)^{1/2} (a^{\dagger} u + u^{\dagger} a)\,. \end{aligned} \tag{9}$$

From Eq. (6) we obtain

$$\begin{aligned} N_d = {} & \eta_1 \eta_2 N_c + (1 - \eta_2) N_v \\ & + \eta_1^{1/2} \eta_2^{1/2} (1 - \eta_2)^{1/2} (c^{\dagger} v + v^{\dagger} c)\,. \end{aligned} \tag{10}$$

From Eqs. (7), (9), and (10)—with the angle brackets denoting quantum averages—we have

$$\langle N_d \rangle = \eta_1 \eta_2 G \xi \langle N_a \rangle + \eta_1 \eta_2 \overline{n}\,, \tag{11}$$

Let an overbar denote further averaging with respect to the other classical randomness, specifically here the random noise n introduced by the electric amplifier; then we obtain

$$\overline{\langle N_d \rangle} = \eta_1 \eta_2 G \xi \langle N_a \rangle + \eta_1 \eta_2 \overline{n}\,, \tag{12}$$

where $\overline{n}$ is the mean of n. Equation (12) contains a signal component $\eta_1 \eta_2 G \xi \langle N_a \rangle$ and a bias term $\eta_1 \eta_2 \overline{n}$ from the mean noise, as expected.

From Eqs. (7), (9), and (10) we can calculate

$$\begin{aligned} \langle N_d^2 \rangle = {} & \eta_1^2 \eta_2^2 G^2 [\xi^2 \langle N_a^2 \rangle + \xi (1 - \xi) \langle N_a \rangle] \\ & + \eta_1^2 \eta_2^2 (n^2 + 2 n G \xi \langle N_a \rangle) \\ & + \eta_1 \eta_2 (1 - \eta_2)(G \xi \langle N_a \rangle + n)\,, \end{aligned} \tag{13}$$

where we have used the fact that all the modes involved are independent. On further averaging over n, one merely replaces n^2 and n by $\overline{n^2}$ and $\overline{n}$ in Eq. (13). Of the six terms in Eq. (13), the first is the signal fluctuation, the second is the signal partition noise from photodetector quantum efficiency loss, the third is the electronic amplifier additive noise, the fourth is the beat noise between the signal and the amplifier additive noise, and the fifth and the sixth represent the partition noise from the laser quantum efficiency loss of the second type. Thus the noise figure (NF) of the PNA defined as $NF \equiv SNR_{N_a}/SNR_{N_d}$ is given by

$$NF = 1 + \frac{1-\xi}{\xi F_a} + \frac{\overline{n^2}}{G^2\xi^2\langle\Delta N_a{}^2\rangle} + \frac{2\overline{n}}{G\xi F_a} + \frac{1-\eta_2}{\eta_1\eta_2 G\xi F_a} + \frac{1-\eta_2}{\eta_1\eta_2 G^2\xi^2}\,\frac{\overline{n}}{\langle\Delta N_a{}^2\rangle} \tag{14}$$

$$= 1 + \frac{1}{F_a}\left(\frac{1-\xi}{\xi} + \frac{2\overline{n}}{G\xi} + \frac{1-\eta_2}{\eta_1\eta_2 G\xi}\right) + \frac{1}{G^2\xi^2\langle\Delta N_a{}^2\rangle}\left(\overline{n^2} + \frac{1-\eta_2}{\eta_1\eta_2}\overline{n}\right), \tag{15}$$

where the Fano factor $F_a \equiv \langle\Delta N_a{}^2\rangle/\langle N_a\rangle$ and we have used $SNR_{N_d} \equiv \langle\tilde{N}_d\rangle^2/\langle\Delta N_d{}^2\rangle$, with $\langle\Delta N_d{}^2\rangle$ being the total amplifier output photon number fluctuation variance and $\langle\tilde{N}_d\rangle$ being the part of the mean output photon number that is proportional to mean amplifier input photon number [i.e., with the bias term in Eq. (12) removed]. A nearly ideal amplifier is obtained under the conditions that the second and third groups of terms in Eq. (15) are small compared with unit. Typically one can obtain

$$\xi \sim 0.9\,, \tag{16}$$

$$G\xi \gg \overline{n}\,, \tag{17}$$

$$\eta_1\eta_2 G\xi \gg 1\,, \tag{18}$$

$$G^2\xi^2 \gg \overline{n^2}\,, \tag{19}$$

so that a good noise performance PNA is realized this way for input states that are nearly classical or classical, such as in the cases of the coherent states, i.e., for $F_a \simeq 1$, $\langle\Delta N_a{}^2\rangle \simeq \langle N_a\rangle$. In particular, if the electronic amplifier additive noise n is Poisson distributed, then relations (17) and (19) become equivalent because in such a case $\overline{n^2} = \overline{n}(\overline{n}+1)$. As numerical estimates, one may obtain[5] $\eta_1 \sim 0.6$, $\eta_2 \sim 0.9$. With these values of η_1 and η_2, relation (16), and the assumptions $F_a \sim 1$, $\langle N_a\rangle \sim 10$, $\overline{n} \sim 1$, $\overline{n^2} \sim 1$, the NF of Eq. (15) becomes $G \sim 10$, $NF \sim 1.3$; $G \sim 100$, $NF \sim 1.1$. These numbers should be compared with those obtained from a phase-insensitive linear amplifier of the same gain, $NF \sim 2$, $G \gg 1$. It can be seen that significant noise reduction is obtained for this PNA realization. However, for strongly sub-Poissonian inputs, i.e., $F_a \ll 1$, it would be difficult in practice to obtain a good PNA in this manner.

The following important conclusion can be drawn from Eq. (15): for the typical situation of large gain and moderate F_a and $\langle\Delta N_a{}^2\rangle$, the noise performance of this PNA is determined by the photoconductor quantum efficiency ξ. Indeed, from Eq. (15),

$$NF \sim 1 + \frac{1}{F_a}\,\frac{1-\xi}{\xi}\,. \tag{20}$$

Under such conditions, the PNA is not significantly limited by the electronic amplifier noise or by the laser quantum efficiency. This is an interesting result, which is consistent with the general principle[6] that large amplification suppresses all the subsequent noises. Our conclusion still holds even if the electronic amplifier noise strength is proportional to the gain G, as is usually the case if the conditions of relation (17) and (19) are obeyed, which is also usually the case.

Finally we observe that the bandwidth or speed of this PNA is determined by that of the photodetector, the laser response time to the driven current, and the signal propagation time from the input to the output. With high-speed photodetectors and fast diode lasers, it is even possible to operate this PNA at picosecond speeds if the whole system is spatially compact. Note also that this realization offers no advantage as a preamplifier for optical detection because a photodetector has already been used in its realization. Compared with the realization described in Ref. 4, which also employs photodetection but neglects the nonunity photodetection quantum efficiency in the feedback loop, the present realization achieves the ideal limit $NF \sim 1$ in a much easier manner.

References

1. H. P. Yuen, Opt. Lett. **12,** 789 (1987).
2. H. P. Yuen, Phys. Rev. Lett. **56,** 2176 (1986).
3. H. P. Yuen, Phys. Lett. A **113,** 405 (1986).
4. Y. Yamamoto, S. Michida, S. Saito, N. Imoto, T. Yanagawa, M. Kitagawa, and G. Björk, in *Progress in Optics,* E. Wolf, ed. (Pergamon, Oxford, 1990), Vol. XXVIII, pp. 87–179.
5. W. H. Richardson, S. Michida, and Y. Yamamoto, Phys. Rev. Lett. **66,** 2867 (1991).
6. H. P. Yuen, in *Photon and Quantum Fluctuations,* E. R. Pike and H. Walter, eds. (Hilger, London, 1988), pp. 1–9.