Optical switching scheme based on the transmission of coupled gap solitons in nonlinear periodic dielectric media

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We study the propagation of two pulses with orthogonal linear polarizations in a nonlinear periodic dielectric structure with $\chi^{(3)}$ nonlinearity. Specifically, we derive the coupled nonlinear Schrödinger equations and find their solitary-wave solutions in a simple case. We show that two orthogonally polarized pulses can copropagate as a coupled gap soliton through a nonlinear periodic structure, while each pulse alone will be strongly reflected owing to the Bragg reflection. Based on the results, we present a new all-optical switching scheme and investigate its operational characteristics.

Theoretical interest in gap solitons has grown recently owing to their unusual properties. For example, it was found[1,2] that gap solitons can propagate in a nonlinear periodic dielectric medium (NPDM) with a group velocity much slower than the usual group velocity determined by the medium's refractive-index dispersion. In addition, gap solitons also exhibit bistable behavior[3,4] owing to $\chi^{(3)}$ nonlinearity in the NPDM. The existence of stationary solitons in a NPDM was first investigated by Chen and Mills[3] by use of a computer simulation in layer-by-layer calculation. They showed that a NPDM with $\chi^{(3)}$ nonlinearity can transmit an input laser beam even if its frequency is located within a stop gap. Such optical fields propagated in the NPDM are referred to as gap solitons since their envelopes have hyperbolic secant shapes. The gap solitons discovered by Chen and Mills are stationary solitons. Later, de Sterke and Sipe[1] and Christodulides and Joseph[2] showed that it is possible to propagate moving solitons in a NPDM. In particular, de Sterke and Sipe[4] derived a nonlinear Schrödinger equation (NLSE) for the gap solitons and clarified the physics behind them.

In this Letter we study the propagation of two orthogonally polarized solitary modes in a NPDM with $\chi^{(3)}$ nonlinearity. We assume that the nonlinear medium has cross-phase-modulation nonlinearity that couples the two modes. First, we find the coupled NLSE's governing the temporal and spatial evolutions of the two orthogonally polarized modes in the NPDM. We then find their solitary-wave solutions in a simple case. The solitary-wave solutions will be referred to as coupled gap solitons. Based on the transmission of coupled gap solitons in a NPDM, we present a new all-optical switching scheme. Specifically, we make use of our result that a coupled gap soliton is described by two orthogonally polarized $\sqrt{3/5}A$ sech pulses copropagating through a NPDM. By contrast, a single $\sqrt{3/5}A$ sech pulse will be strongly reflected because its amplitude is less than that needed to propagate a gap soliton in a single polarization. As a result, we can realize a light-by-light switch by exploiting this property of the coupled gap soliton. Finally, we will investigate the switching performance with a GaAs waveguide[5] as the NPDM and show the potential promise of the proposed device.

In order to study the dynamic behavior of the coupled gap solitons, we use an envelope-function approach[4] to derive the coupled NLSE's in a NPDM. Following Refs. 4 and 6, the coupled NLSE's are derived by expressing the electric field $E(z, t)$ in terms of different length scales, $z_n = p^n z$, and time scales, $t_n = p^n t$: $\mathbf{E}(z, t) = p(e_{1x}\hat{x} + e_{1y}\hat{y}) + p^2(e_{2x}\hat{x} + e_{2y}\hat{y}) + \ldots$. Let a_x and a_y be the slowly varying envelopes of two pulses with orthogonal polarizations. Substituting $\mathbf{E}(z, t)$ into the Maxwell equations and then calculating order by order for p^n, one can obtain the coupled nonlinear differential equations by combining each of the results for $n = 1$, 2, and 3. The coupled nonlinear differential equations governing temporal evolutions of two envelopes are given as follows:

$$\begin{aligned} i\frac{\partial a_x}{\partial t} + \frac{1}{2}\omega''_{mx}\frac{\partial^2 a_x}{\partial z^2} + (\alpha_{mxx}|a_x|^2 + \alpha_{mxy}|a_y|^2)a_x \\ + \beta_{mxy}{a_y}^2 a_x^* \exp[2i(\omega_{mx} - \omega_{my})t] = 0, \\ i\frac{\partial a_y}{\partial t} + \frac{1}{2}\omega''_{my}\frac{\partial^2 a_y}{\partial z^2} + (\alpha_{myy}|a_y|^2 + \alpha_{myx}|a_x|^2)a_y \\ + \beta_{myx}{a_x}^2 a_y^* \exp[2i(\omega_{my} - \omega_{mx})t] = 0, \end{aligned} \tag{1}$$

where $\alpha_{mxx} = 6\pi\omega_{mx}{}^2 L \int_0^L \chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^4 \mathrm{d}z_0$, $\alpha_{mxy} = 4\pi\omega_{mx}{}^2 L \int_0^L \chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^2|\varphi_{my}(z_0)|^2 \mathrm{d}z_0$, $\alpha_{myx} = 4\pi\omega_{my}{}^2 L \int_0^L \chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^2|\varphi_{my}(z_0)|^2 \mathrm{d}z_0$, $\beta_{mxy} = 2\pi(\omega_{mx} - 2\omega_{my})^2 L \int_0^L \chi^{(3)}(z_0)\varphi_{mx}{}^2(z_0)\varphi_{my}^{*2}(z_0)\mathrm{d}z_0$, ω''_{mx} is the curvature of the photonic band, and φ_{mx} and φ_{my} are the fast-varying Bloch functions along the x and y axes, respectively. The cross-coupling coefficients $\chi^{(3)}_{abcd}$ are related to the self-coupling coefficient $\chi^{(3)}$ by $\chi^{(3)}_{xxyy} = \chi^{(3)}_{xyxy} = \chi^{(3)}_{xyyx} = (1/3)\chi^{(3)}$. In this Letter we deal mainly with the case in which the last terms (called the analogous four-wave-mixing terms) in Eqs. (1) can be neglected. It is valid if the x- and y-polarized fields have different center

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frequencies (or if the fields are in a birefringent medium).[7] In this case, Eqs. (1) are reduced to the coupled NLSE's. As discussed below, in the case when the terms of concern are not negligible, simple solutions can be found if $\omega_{mx} = \omega_{my}$.

The coupled NLSE's can be solved in the special case of stationary solitons by introduction of the detuning factors δ_x and δ_y, where $a_x = \psi_x(z)\exp(-i\delta_x t)$ and $a_y = \psi_y(z)\exp(-i\delta_y t)$. In terms of these detuning factors, Eqs. (1) become the following equations:

$$\frac{d^2\psi_x}{dz^2} - B_x{}^2\psi_x + 2\frac{B_x{}^2}{A_{xx}{}^2}|\psi_x|^2\psi_x + 2\frac{B_x{}^2}{A_{xy}{}^2}|\psi_y|^2\psi_x = 0,$$

$$\frac{d^2\psi_y}{dz^2} - B_y{}^2\psi_y + 2\frac{B_y{}^2}{A_{yy}{}^2}|\psi_y|^2\psi_y + 2\frac{B_y{}^2}{A_{yx}{}^2}|\psi_x|^2\psi_y = 0, \tag{2}$$

where $A_{xx} = (-2\delta_x/\alpha_{mxx})^{1/2}$, $A_{xy} = (-2\delta_x/\alpha_{mxy})^{1/2}$, $A_{yy} = (-2\delta_y/\alpha_{myy})^{1/2}$, $A_{yx} = (-2\delta_y/\alpha_{myx})^{1/2}$, $B_x = (-2\delta_x/\omega''_{mx})^{1/2}$, and $B_y = (-2\delta_y/\omega''_{my})^{1/2}$.

First let us consider a special case in which the two orthogonal fields have the same shape but not necessarily the same amplitude, so that $a_x = \psi_x \exp(-i\delta t) \equiv \psi \exp(-i\delta t)$ and $a_y = K\psi \exp(-i\delta t)$. We also assume that $B_x = B_y$. In this case, the coupled NLSE's are simplified to two NLSE's of the same form. These two NLSE's can then be solved by following similar methods used in Refs. 3 and 4. The pulse envelope solutions are given by the following solitary waveforms:

$$\psi_x(z) = \psi(z) = \frac{1}{(1/A_{yx}{}^2 + K^2/A_{yy}{}^2)^{1/2}}\,\mathrm{sech}(Bz), \tag{3}$$

$$\psi_y(z) = K\psi(z) = \frac{K}{(1/A_{xx}{}^2 + K^2/A_{xy}{}^2)^{1/2}}\,\mathrm{sech}(Bz), \tag{4}$$

where K is given by $K^2 = (1/A_{xx}{}^2 - 1/A_{yx}{}^2)/(1/A_{yy}{}^2 - 1/A_{xy}{}^2)$. If we assume that the medium and pulse characteristics are identical for TE and TM modes with a slightly different frequency, then $\varphi_{mx} \simeq \varphi_{my}$, $\omega_m \equiv \omega_{mx} \simeq \omega_{my}$, $\omega''_{mx} \simeq \omega''_{my}$, and thus $\alpha_m \equiv \alpha_{mxx} \simeq \alpha_{myy} \simeq (3/2)\alpha_{mxy}$. In this case, the envelope functions of gap solitons can be further simplified, giving $a = a_x = a_y = \sqrt{3/5}A\,\mathrm{sech}(Bz)\exp(-i\delta t)$, where $A = \sqrt{-2\delta/\alpha_m}$ and $B = \sqrt{-2\delta\omega''_m}$. In the general case of moving solitons, the solutions give[1,8]

$$a = a_x = a_y = \sqrt{3/5}A\exp(iB_2 z)\exp(-i(\delta + \Delta)t) \times \mathrm{sech}\, B_1(z - v_g t), \tag{5}$$

where $A = \sqrt{-2\delta/\alpha_m}$, $B_1 = \sqrt{-2\delta/\omega''_m}$, $B_2 = \sqrt{2\Delta/\omega''_m}$, $v_g = \sqrt{2\Delta\omega''_m}$, and the center frequency of the soliton is $\omega_c = \omega_m + \delta + \Delta$. The factor $\delta + \Delta$ corresponds to the frequency detuning of the pulse center frequency from the edge of the stop band with frequency ω_m.

We note that our solution can be straightforwardly extended to the case in which the analogous four-wave-mixing terms are not negligible, provided that $\omega_{mx} = \omega_{my}$. In this case, there is also a coupled soliton solution, where a_x is proportional to a_y. In fact, with a_x proportional to a_y, the analogous four-wave-mixing terms would simply give rise to an effective change of coefficients of the bracketed terms in Eqs. (1). For example, the case discussed above, where $a_x = a_y$, would have a solution of $a = a_x = a_y = (1/\sqrt{2})A\,\mathrm{sech}(Bz)\exp(-i\delta t)$ instead.

Based on these results, we propose a new all-optical switching scheme. This scheme is different from the intensity-dependent switch studied in the literature.[9] In our proposed switching scheme, we make use of the fact from Eq. (5) that two orthogonal $\sqrt{3/5}A$ sech pulses can copropagate as coupled gap solitons owing to cross-coupling $\chi^{(3)}$ nonlinearity, although one $\sqrt{3/5}A$ sech pulse will be strongly reflected. A single $\sqrt{3/5}A$ sech pulse will not propagate through the medium, because in the formalism here the condition required for propagation of a gap-soliton pulse in a single polarization is given by a pulse with amplitude A, i.e., a A sech pulse. Thus a $\sqrt{3/5}A$ sech pulse with an amplitude smaller than A will be strongly reflected. This proposed light-by-light switch is shown in Fig. 1. The main part of the device consists simply of an extremely small switching element, i.e., a GaAs waveguide, and two polarization beam splitters (PBS's). In the pulse generation part, a $\sqrt{6/5}A$ sech pulse is divided by a PBS into two $\sqrt{3/5}A$ sech pulses that are orthogonally polarized with respect to each other. The two synchronized pulses (the signal S and the control C) are then combined at another PBS and coupled into the GaAs waveguide. S will be transmitted through the waveguide only with the presence of C owing to the characteristic of coupled gap solitons. S then is separated from the coupled state by another PBS at the waveguide output.

We have done some numerical calculations to look at the feasibility of the proposed switch. A good candidate for the NPDM is a GaAs waveguide, which was demonstrated by Ho *et al.* to have a strong $\chi^{(3)}$ nonlinearity at 1.55–1.65 μm.[5] From their experimental data, we take the nonlinear refractive index of GaAs at 1.6 μm to be $n_2 = 3.6 \times 10^{-14}$ cm^2/W. We assume that the waveguide has a Bragg structure with a period of $d = 0.23$ μm, giving a Bragg wavelength of $\lambda_0 = 1.6$ μm ($n \simeq 3.5$ for GaAs). The stop-gap bandwidth $\Delta\omega$ is determined by the Bragg structure's refractive-index modulation depth (Δn). Assuming that $\Delta n/\bar{n} = 0.01$, we have $\Delta\omega = 1.178 \times 10^{13}$/s. In this case, the pulse width is restricted to be larger

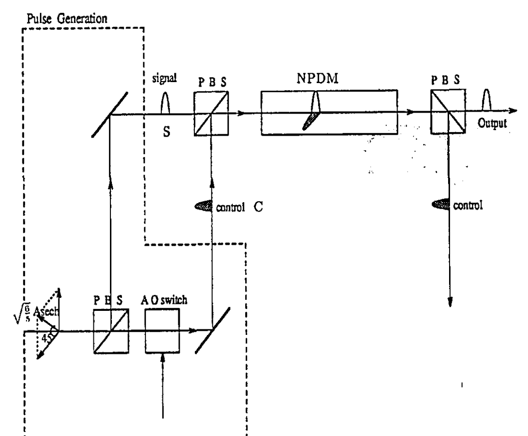

Fig. 1. Schematic of light-by-light switching based on the transmission of coupled gap solitons. AO, acousto-optical.

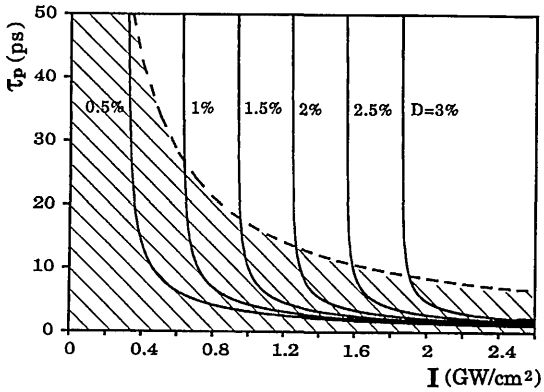

Fig. 2. Dependence of a temporal width on the intensity for each fixed normalized detuning D. The dashed curve indicates a limitation of pulse bandwidth in conjunction with the center frequency and intensity.

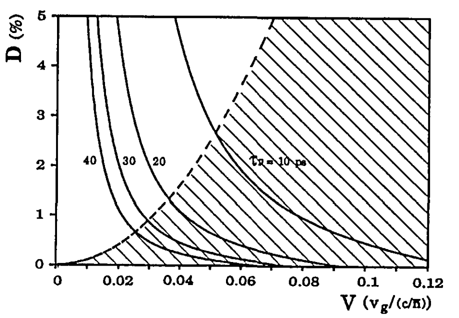

Fig. 3. Dependence of a normalized velocity $v_g/(c/\bar{n})$ on the normalized detuning D for each temporal width. The dashed curve indicates a limitation of pulse bandwidth in conjunction with temporal width and velocity.

than 533 fs if the entire pulse spectrum is to be within the stop-gap bandwidth.

By using the above parameters, we first investigate the pulse width τ_p of the coupled gap soliton as a function of its peak intensity I in each polarization. I is defined as the pulse intensity inside the Bragg structure. The dependence of τ_p on I would be different for different frequency detuning from the edge of the stop band. Let us define a normalized frequency detuning factor D by $D \equiv (|\delta + \Delta|)/\Delta\omega = (|\omega_c - \omega_m|)/\Delta\omega$. The numerical results for the dependence of τ_p on I are shown in Fig. 2, in which a family of curves is generated with different D values. For good switching performance, we require that ω_c be far enough from the edge that the entire pulse spectrum is within the stop band. Let us call this requirement the detuning condition. The region in which the detuning condition is satisfied is the unshaded portion of Fig. 2 (to the upper right-hand side of the dashed curve). We note that much higher input power[10] than I would be required for the generation of gap solitons because of the reflections at the interfaces between the Bragg grating region and the surrounding region. However, this power loss owing to reflections can be reduced by use of the index-matching methods suggested recently by de Sterke[10] and Haus.[11]

In addition to the dependence of pulse width on pulse intensity and pulse frequency detuning, it turns out that the group velocity v_g of the pulse is also a strong function of the pulse parameters. The dependence of v_g on D for various pulse widths is plotted as a family of curves in Fig. 3. We assume that the medium is nondispersive, with the usual group velocity given by $c/\bar{n}$. The value of v_g is normalized by $c/\bar{n}$. Again, the unshaded portion of the figure indicates the region in which the detuning condition is satisfied.

As an example, we see from Fig. 2 that if we have a laser with 20-ps pulses, then a coupled gap soliton of 20-ps width can be generated with an intensity greater than 1.0 GW/cm^2 and a normalized frequency detuning greater than 1.5% from the edge of the stop band. From Fig. 3, we then see that the group velocity of the coupled gap soliton can range from 0.037 to 0.02 of $c/\bar{n}$ by adjustment of the normalized frequency detuning from 1.5% to 4.5%. Note that because of the slow group velocity, this 20-ps coupled gap soliton is only approximately 0.03–0.06 mm long in space for the above frequency detuning range.

An important factor governing the switch is the switching contrast ratio, defined as $Q = (\text{Power}_{\text{off state}}/\text{Power}_{\text{on state}})$. When the control pulse is off, the single $\sqrt{3/5}A$ sech pulse will decay exponentially in NPDM with a decay length given approximately by $1/B_1$ [see Eqs. (2)]. By using this, we can obtain an estimate for the value of Q. For example, if the medium length is 1 mm, then Q can be as small as $Q \leq 10^{-7}$ for $I = \sim 0.8$–1.5 GW/cm^2. This 1-mm length corresponds to 4348 periods of waveguide corrugations (stacks). This is more than enough periods for the formation of gap solitons as given by the N parameter in Ref. 4.

In conclusion, we have proposed a compact all-optical switch and also investigated its feasibility. Its implementation is simple because the main part consists simply of two PBS's and a switching element (GaAs waveguide) and is thus compact in size. Moreover, the proposed device can potentially exhibit good switching performance, including good switching speed with reasonable pulse width, and good on–off intensity contrast ratio.

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