Joint Optimization of MAC and Network Coding for Cooperative and Competitive Wireless Multicasting

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Abstract

In this paper, we address the problem of cross-layer optimization in medium access control (MAC) and network layers for wireless multicasting with multiple cooperative or competitive source nodes in a simple tandem network. We consider scheduled or random access in MAC layer and model network layer operations as network coding or plain routing. We separately look at the cooperative and competitive operation with total and individual performance objectives. We evaluate the resulting cross-layer interactions between MAC and network layers and specify throughput optimization trade-offs intertwined with energy efficiency objectives. We follow a game-theoretic analysis to point at the inefficiency of the non-cooperative equilibrium with selfish nodes competing for limited network resources. In this context, we introduce distributed cooperation stimulation mechanisms to improve the non-cooperative network operation to the cooperative equilibrium performance.

1. Introduction

MAC and network layers are interdependent in ad hoc wireless networks and need to be jointly specified. The MAC layer operations coordinate transmissions of nodes for reliable communication either in random or scheduled access form, whereas the network layer operations of network coding or plain routing deliver packets from sources to destinations. Network coding was originally developed for wired networks with simultaneous multiple transmissions and receptions over different links (without interference effects). The main performance focus has been the multicast throughput rate that is optimized by network coding to the Max-flow Min-cut bound [1].

Wireless network operation incorporates multiple performance criteria of throughput and energy efficiency, and involves different optimization trade-offs that have not been fully developed or clearly understood yet. Network coding has been extended in [2]-[5] to operate in wireless networks with additional properties of omnidirectional transmissions, interference effects and single transceiver per node. The problem of deriving network codes in conjunction with conflict-free scheduling has been addressed in [2] for wireless multicasting with a single source. The extension to multiple sources has been outlined in [5] by jointly specifying throughput rates achievable by different source nodes.

For efficient performance evaluation in wireless networks, throughput measures need to reflect the total and minimum throughput rates over all source-destination pairs, whereas energy properties need to incorporate both transmission and processing costs. It is necessary to formulate a cross-layer optimization framework for throughput and energy efficiency in MAC and network layers.

In this paper, we first specify the achievable throughput region as the constraint set of cooperative optimization in tandem networks. We evaluate scheduled or random access operations in MAC layer and network coding or plain routing operations in network layer as solutions to the problem of cross-layer throughput and energy optimization.

The previous (wired or wireless) network coding studies rely on cooperation of nodes to jointly optimize network performance objectives. However, it is difficult to coordinate a large number of nodes under cooperation-based MAC and network coding (or plain routing). If selfish nodes with individual performance objectives compete for MAC and network layer tasks, they are subject to performance loss compared to centralized cooperation [6]. The cooperation can be realized externally by a central authority, or we can impose distributed cooperation reinforcement mechanisms and let nodes operate selfishly in a distributed manner to optimize the individual performance measures involving throughput and (transmission and processing) energy costs.

Our objective is to extend the analysis to non-cooperative operation with nodes competing for limited network resources of bandwidth and energy. We formulate a joint MAC and network coding game with node utilities reflecting individual throughput and energy efficiency objectives.

We introduce distributed cooperation stimulation mechanisms and evaluate the improvement in non-cooperative equilibrium strategies of MAC and network coding (or plain routing). First, we consider the suboptimal Tit-for-Tat-based punishment mechanism based on mimicking strategies of neighbor nodes. Next, we introduce a pricing mechanism to charge credits for packet relaying. We show that it is possible to let nodes make selfish MAC and network coding (or plain routing) decisions in a distributed fashion while approaching performance results under centralized cooperation. We specify how the non-cooperative equilibrium strategies of network coding or plain routing depend on the throughput credits, energy costs and relay rewards.

The paper is organized as follows. We present the wireless network model in section 2 and formulate the cross-layer optimization problem for scheduled and random access in section 3. We discuss throughput optimization in section 4 and incorporate energy efficiency measures in section 5. We extend the analysis to non-cooperative network operation in section 6 and draw conclusions in section 7.

2. Wireless Network Model

We consider a linear tandem network with node set N, as shown in Figure 1. We assume that the transmission time of each packet is one time slot, and consider multihop packet propagation in a store-and-forward manner instead of continuous information flows. We define $\lambda_{i,j}$ as the achievable throughput rate from source node i to destination node j in multicast group M_i. We consider multicast communication, i.e. $\lambda_{i,j} = \lambda_i, i \in N, j \in M_i$. We assume saturated packet queues, whereas the case of possibly emptying queues has been discussed in [5] for wireless network coding.

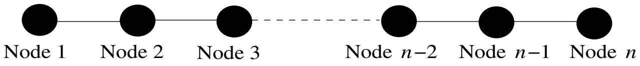

Figure 1. Linear tandem network model.

We assume omnidirectional transmissions of at most one packet per time slot and single transceiver per node. Since nodes cannot simultaneously transmit and receive packets, they need to be divided into disjoint sets of transmitters and receivers in every time slot. We consider the classical collision channel model such that a packet transmission is successful, if it is the only transmission that reaches the intended receiver. We separately consider conflict-free transmission schedules and random access in MAC layer.

(a) Scheduled Access: We order n nodes from left to right and divide them into three groups such that node i is included in group $m = (i-1)(\mathrm{mod}\ 3) + 1$, where $1 \leq m \leq 3$. Nodes in group m are activated for disjoint time fraction t_m, where $0 \leq t_m \leq 1$ and $\sum_{m=1}^{3} t_m = 1$.

(b) Random Access: Each node i transmits a packet at any time slot with fixed probability p_i. The collided packets remain backlogged until they are successfully received. A transmission randomly carries a source or a relay packet.

Each node i has three separate queues of infinite capacities: Queue Q_i^1 stores source packets it generates and two separate queues Q_i^2 and Q_i^3 store relay packets that are incoming from its right and left neighbors, respectively. Each packet coming to node i from one of its neighbors must be transmitted to the neighbor on the other side. In network layer, we consider plain routing or network coding.

(a) Plain Routing: Node i either transmits a packet from source queue Q_i^1 or a packet from one of its relay queues Q_i^2 and Q_i^3 that can be combined to a single queue (in a first-come-first-served fashion).

(b) Network Coding: Node i either transmits a packet from source queue Q_i^1 or a linear combination of two packets, one from each of the relay queues Q_i^2 and Q_i^3. We consider each packet as a vector of bits and assume F_2 as the field for linear network coding operations such that the bitsum $x + y$ of two packets x and y is a modulo-2 vector addition of corresponding vectors of each packet. We can separate the transmissions of source and relay packets without performance loss of throughput or energy efficiency.

3. Cooperative Network Optimization

We assume node cooperation and specify the constraints on the achievable throughput rates $\underline{\lambda} = \lambda_{i,j}, i \in N, j \in M_i$, separately for network coding and plain routing only. These constraints determine the achievable throughput region $\mathcal{A}$ as function of the transmission schedules $\underline{t} = \{t_m\}_{m=1}^{3}$ or transmission probabilities $\underline{p} = \{p_i\}_{i \in N}$ under scheduled access or random access, respectively. The dependence of optimization constraints (namely region $\mathcal{A}$) on $\underline{t}$ or $\underline{p}$ leads to cross-layer optimization in MAC and network layers.

We use two aggregate performance criteria of the sum-delivered throughput $\lambda_\Sigma = \sum_{i \in N} |M_i| \lambda_i$ and minimum transmitted throughput $\lambda_{\min} = \min_{i \in N} \lambda_i$ to represent the total and minimum throughput values. We will introduce the energy efficiency measures in section 5. The cross-layer optimization problem is formulated as:

$$\text{Select } \underline{\lambda} \in \mathcal{A} \text{ and specify } \underline{t} \text{ or } \underline{p} \text{ to maximize } \lambda_\Sigma \text{ or } \lambda_{\min}$$

3.1. Constraints under scheduled access

We define N_i^r as the set of nodes with packets that arrive at node i from the right direction and need to be forwarded to the left neighbor of node i, and we define N_i^l as the set of nodes with packets that arrive at node i from the left direction and need to be forwarded to the right neighbor of node i. Let $\Lambda_i^r = \sum_{j \in N_i^r} \lambda_j$ and $\Lambda_i^l = \sum_{j \in N_i^l} \lambda_j$ denote the rate of relay traffic incoming from the right and left neighbor nodes of node i, respectively.

Theorem 1. *The achievable throughput region $\mathcal{A}$ with $\lambda_i \geq 0, i \in N$, is given by*

$$\sum_{m=1}^{3} \max_{i:m(i)=m} [\lambda_i + \max(\Lambda_i^r, \Lambda_i^l)] \leq 1 \qquad (1)$$

$$\sum_{m=1}^{3} \max_{i:m(i)=m} [\lambda_i + \Lambda_i^r + \Lambda_i^l] \leq 1 \qquad (2)$$

for network coding and plain routing, respectively.

Proof. Each node i separately transmits packets it generates and (plain or coded) relay packets for τ_i and $1-\tau_i$ fractions of time (whenever it is scheduled to transmit), respectively. The achievable throughput rates $\underline{\lambda}$ satisfy

$$0 \leq \lambda_i \leq t_{m(i)}\tau_i, \Lambda_i^r \leq t_{m(i)}(1-\tau_i), \Lambda_i^l \leq t_{m(i)}(1-\tau_i)$$

$$0 \leq \lambda_i \leq t_{m(i)}\tau_i,\ \lambda_i + \Lambda_i^r + \Lambda_i^l \leq t_{m(i)}(1-\tau_i)$$

for network coding and plain routing, respectively, where $m(i) = (i-1) \pmod 3 + 1$, $i \in N$. For any time fraction τ_i, the achievable throughput rates $\lambda_i \geq 0, i \in N$, satisfy

$$\lambda_i + \Lambda_i^r \leq t_{m(i)}, \qquad \lambda_i + \Lambda_i^l \leq t_{m(i)} \qquad (3)$$

$$\lambda_i + \Lambda_i^r + \Lambda_i^l \leq t_{m(i)} \qquad (4)$$

for network coding and plain routing, respectively. We sum up the traffic loads over all disjoint time fractions t_1, t_2 and t_3 to obtain the region $\mathcal{A}$ that imposes only linear constraints (1) and (2) that are independent of schedules $\underline{t}$. ☐

We can also consider a hybrid network, in which nodes in group $N_R \subseteq N$ are limited to plain routing (e.g. due to hardware restriction), whereas the rest of nodes are capable of network coding. The resulting constraints on $\mathcal{A}$ are

$$\sum_{m=1}^{3} \max \Big(\max_{i \notin N_R : m(i)=m} [\lambda_i + \max(\Lambda_i^r, \Lambda_i^l)], \max_{i \in N_R : m(i)=m} [\lambda_i + \Lambda_i^r + \Lambda_i^l] \Big) \ \leq\ 1$$

In section 6, we will consider the case when any selfish node i chooses between plain routing or network coding (i.e. $i \in N_R$ or not) or discarding relay traffic (i.e. $\tau_i = 1$, $\Lambda_i^r = 0$, $\Lambda_i^l = 0$) to optimize the individual performance.

3.2. Constraints under random access

We assume that every node randomizes between transmitting source and relay packets. We consider three transmission methods for source packets. In method A, nodes transmit new source packets without channel feedback. Only reception of a packet by all intended neighbors contributes to throughput. Method B uses channel feedback and allows nodes to transmit a new source packet only if the previous packet has been received by all intended neighbors. In method C, each node transmits linear combination of source packets that have not been decoded yet by neighbors. If transmission of packet x by node i is only received by $i+1$, node i transmits $x+y$ (instead of x as in method B). If $x+y$ is successfully received, node i needs to deliver y only to $i-1$ (rather than to both $i-1$ and $i+1$ as in method B). Define s_i^r and s_i^l as the probability that a transmission of node i is successfully received by the right and left neighbor, respectively, i.e. $s_i^r = (1-p_{i+1})(1-p_{i+2})$ for $1 \leq i \leq n-2$, $s_{n-1}^r = 1 - p_n$, $s_n^r = 1$, $s_i^l = (1-p_{i-2})(1-p_{i-1})$ for $3 \leq i \leq n$, $s_1^l = 1$ and $s_2^l = 1 - p_1$. The achievable throughput rates $\lambda_i, i \in N$, satisfy

$$s_i^l \lambda_i + \gamma_i(s_i^r, s_i^l)\Lambda_i^r \leq p_i s_i^l \gamma_i(s_i^r, s_i^l)$$
$$s_i^r \lambda_i + \gamma_i(s_i^r, s_i^l)\Lambda_i^l \leq p_i s_i^r \gamma_i(s_i^r, s_i^l)$$

for network coding and

$$s_i^r s_i^l \lambda_i + s_i^r \gamma_i(s_i^r, s_i^l)\Lambda_i^r + s_i^l \gamma_i(s_i^r, s_i^l)\Lambda_i^l \leq p_i s_i^l s_i^r \gamma_i(s_i^r, s_i^l)$$

for plain routing, where $\gamma_i(s_i^r, s_i^l)$ is defined as $s_i^r s_i^l$, $\frac{{s_i^l}^2 {s_i^r}^2 + {s_i^l}^2 s_i^r (1-s_i^r) + {s_i^r}^2 s_i^l (1-s_i^l)}{s_i^l s_i^r + {s_i^l}^2 (1-s_i^r) + {s_i^r}^2 (1-s_i^l)}$ and $\min(s_i^r, s_i^l)$ for method A, B and C, respectively. The region $\mathcal{A}$ is a non-linear function of $\underline{p}$ and optimized by method C.

4. Optimization of Throughput Objectives

First, we consider linear optimization with linear constraints in scheduled access. For broadcast communication, i.e. $M_i = N - \{i\}$, the optimal throughput rates in terms of λ_Σ and $\lambda_{\min}$ are shown in Figure 2. The throughput rates for optimal $\lambda_{\min}$ approach each other under network coding and plain routing, as n increases, and can only reach half of the optimal values of λ_Σ under network coding. The objectives of maximizing λ_Σ and $\lambda_{\min}$ cannot be achieved simultaneously, e.g. $\lambda_{\min}$ is limited to 0 for the optimal solutions of λ_Σ. We can also maximize λ_Σ subject to $\lambda_{\min} \geq \alpha$ for some positive constant α, i.e. $\lambda_{i,j} \geq \alpha$ for all $i \in N$ and $j \in M_i$. We solve the resulting optimization problem by the Lagrange Multipliers method. The optimal value of λ_Σ is illustrated in Figure 3 as function of α.

Next, we consider unicast communication, i.e. $|M_i| = 1$, $i \in N$. The best and least favorable unicast demands choose each destination as the one-hop neighbor of the source and the node that has the largest hop distance from the source, respectively. We also consider uniform demand such that the destination of each source is randomly chosen. We evaluate in Figure 4 throughput rates for optimal time allocation and suboptimal equal time allocation $t_m = \frac{1}{3}, m = 1, 2, 3,$

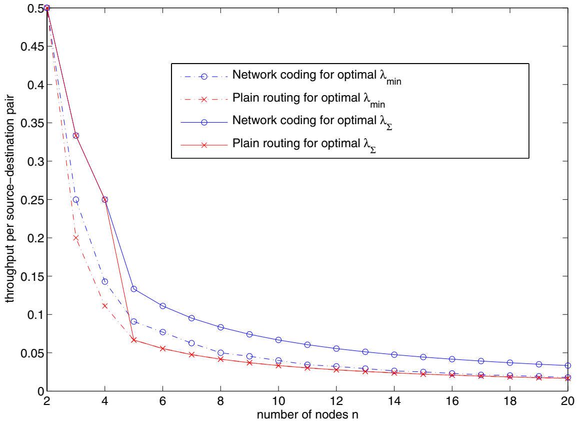

Figure 2. Achievable throughput for broadcast communication in scheduled access.

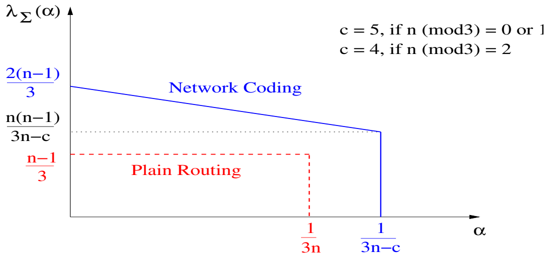

Figure 3. The optimal value of λ_Σ for $\lambda_{\min} \geq \alpha$.

and compare them to bounds imposed by the best and least favorable demands. The results highlight the interdependence of MAC and network layers. As n goes to infinity, network coding doubles the value of λ_Σ compared to plain routing. Depending on whether we consider unicast or broadcast communication, network coding achieves twice or same value of $\lambda_{\min}$ as plain routing. This underlines the trade-offs depending on communication demands.

Next, we address the problem of maximizing λ_Σ in random access. The constraints on $\mathcal{A}$ are non-linear functions of $\underline{p}$. The achievable throughput rates per source-destination pair are depicted in Figure 5 for broadcast communication.

5. Extensions to Energy Efficiency Objectives

First, we consider transmission energy cost $E_t(\underline{\lambda})$ per time slot to achieve throughput rates $\underline{\lambda} \in \mathcal{A}$. Let $\mathcal{E}_t$ denote the energy cost of each transmission. Node i transmits source packets with rate λ_i incurring cost $\mathcal{E}_t \lambda_i$ per time slot. Node i receives relay packets with rates Λ_i^r and Λ_i^l from the right and left neighbors. For network coding, node i consumes $\mathcal{E}_t$ amount of energy to transmit one coded packet from both relay queues Q_i^2 and Q_i^3. Since the relay packets arrive at queues Q_i^2 and Q_i^3 with rates Λ_i^r and Λ_i^l, respectively, the total energy per time slot consumed to relay packets is $\mathcal{E}_t \max(\Lambda_i^r, \Lambda_i^l)$ for network coding and we obtain $E_t(\underline{\lambda}) = \sum_{i=1}^n \mathcal{E}_t(\lambda_i + \max(\Lambda_i^r, \Lambda_i^l))$. For plain routing, the total energy consumption per time slot to transmit relay packets is $\mathcal{E}_t(\Lambda_i^r + \Lambda_i^l)$ and we obtain $E_t(\underline{\lambda}) = \sum_{i=1}^n \mathcal{E}_t(\lambda_i + \Lambda_i^r + \Lambda_i^l)$.

Next, we consider processing energy cost $E_p(\underline{\lambda})$ per time slot to achieve throughput rates $\underline{\lambda} \in \mathcal{A}$. Let $\mathcal{E}_c$ and $\mathcal{E}_f$ denote the energy cost of a coding or decoding operation (namely energy cost of binary vector addition) and plain forwarding, respectively. For network coding, $E_p(\underline{\lambda}) = \sum_{i=1}^n 3\mathcal{E}_c \min(\Lambda_i^r, \Lambda_i^l) + \mathcal{E}_f(\max(\Lambda_i^r, \Lambda_i^l) - \min(\Lambda_i^r, \Lambda_i^l))$,

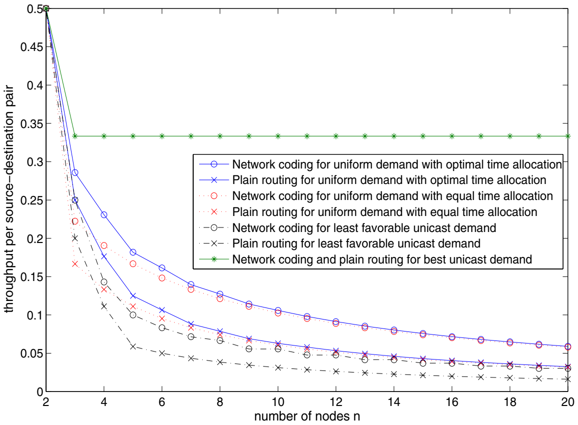

Figure 4. Achievable throughput for unicast communication in scheduled access.

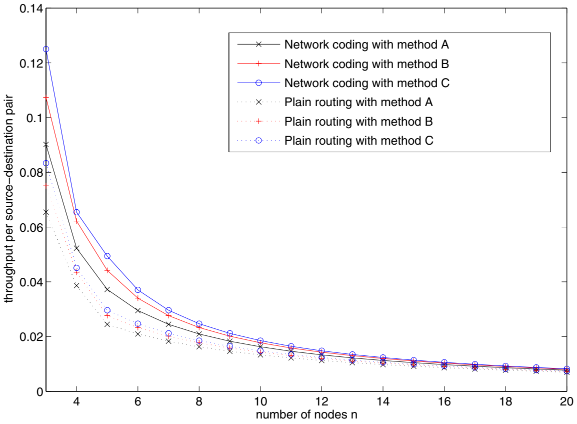

Figure 5. Achievable throughput for broadcast communication in random access.

since relay node i performs network coding with rate $\min(\Lambda_i^r, \Lambda_i^l)$ and each coding operation is accompanied with two decoding operations at neighbors, whereas the rest of relay packets are forwarded with cost $\mathcal{E}_f$. For plain routing, $E_p(\underline{\lambda})=\sum_{i=1}^n \mathcal{E}_f(\Lambda_i^r+\Lambda_i^l)$, since node i forwards relay packets with rate $\Lambda_i^r + \Lambda_i^l$. Network coding reduces total (transmission and coding) energy cost per packet compared to plain routing, if $3\mathcal{E}_c < \mathcal{E}_t + 2\mathcal{E}_f$, where $\mathcal{E}_c$ and $\mathcal{E}_f$ depend on queue management and hardware constraints. We cannot optimize throughput and energy costs simultaneously.

6. Competitive Network Operation

We assume that each node i is rational and has the selfish objective of maximizing its own utility function u_i. The strategy space of node i is the set $\lambda_i^j, j \in N$, where λ_i^j is defined as the rate at which node i transmits packets of node j such that $\Lambda_i^r = \sum_{j \in N_i^r} \lambda_i^j$ and $\Lambda_i^l = \sum_{j \in N_i^l} \lambda_i^j$. The utility of node $i \in N$ is defined as function of $\lambda_j^i, j \in N$, to represent the total multicast throughput $|M_i|\lambda_i$, transmission energy cost $E_{t,i}$ and processing energy cost $E_{p,i}$ as

$$u_i(\underline{\lambda}_i, \underline{\lambda}_{-i}) = |M_i|\lambda_i - E_{t,i}(\underline{\lambda}) - E_{p,i}(\underline{\lambda}) \qquad (5)$$

where $\underline{\lambda}_i = \{\lambda_i^j\}_{j \in N}$ and $\underline{\lambda}_{-i} = \{\underline{\lambda}_j, j \in N - \{i\}\}$. The components of u_i are $\lambda_i = \min(\lambda_i^i, \{\lambda_j^i\}_{j \in R_i})$, where R_i is the set of relay nodes for i, $E_{t,i}(\underline{\lambda}) = \mathcal{E}_t(\lambda_i^i + \max(\Lambda_i^r, \Lambda_i^l))$ for network coding, $E_{t,i}(\underline{\lambda}) = \mathcal{E}_t(\lambda_i^i + \Lambda_i^r + \Lambda_i^l)$ for plain routing, $E_{p,i}(\underline{\lambda}) = \sum_{k \in \{-1,0,1\}} \mathcal{E}_c \min(\Lambda_{i+k}^r, \Lambda_{i+k}^l) + \mathcal{E}_f(\max(\Lambda_i^r, \Lambda_i^l) - \min(\Lambda_i^r, \Lambda_i^l))$ for network coding, and $E_{p,i}(\underline{\lambda}) = \mathcal{E}_f(\Lambda_i^r + \Lambda_i^l)$ for plain routing. We define the best response function of node i as $\underline{b}_i(\underline{\lambda}_{-i}) = \operatorname{argmax}_{\underline{\lambda}_i} u_i(\underline{\lambda}_i, \underline{\lambda}_{-i})$. Then, $\underline{\lambda}_i^*, i \in N$, is a Nash equilibrium strategy set (such that no node can unilaterally improve its utility, if the strategies of other nodes remain the same), if and only if $\underline{\lambda}_i^* \in \underline{b}_i(\underline{\lambda}_{-i}^*)$ for all $i \in N$.

Theorem 2. *(a) For zero energy costs $\mathcal{E}_t$, $\mathcal{E}_c$ and $\mathcal{E}_f$, any achievable set of rates $\lambda_i, i \in N$, results in a non-cooperative Nash equilibrium strategy.*

(b) For $\mathcal{E}_t > 0$, $\mathcal{E}_c > 0$ or $\mathcal{E}_f > 0$, the unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = 0$ for any $i \in N$, $j \in N - \{i\}$, and $\lambda_i^i = 0$, if $|R_i| > 0$ or $|R_i| = 0$ and $\mathcal{E}_t > |M_i|$. Otherwise, $\lambda_i^i = \frac{1}{2}$ for $n{=}2$ and $\frac{1}{3}$ for $n \geq 3$.

Proof. (a) For zero energy costs $\mathcal{E}_t$, $\mathcal{E}_c$ and $\mathcal{E}_f$, the best response of node i given any $\underline{\lambda}_{-i}$ is to set $\lambda_i^i \geq \lambda_j^i$ for $j \in R_i$. However, λ_i is upper bounded by λ_j^i for $j \in R_i$ and therefore throughput rates $\lambda_j \in \mathcal{A}$ directly impose throughput rate λ_i that cannot be improved individually by node i. Therefore, any achievable set of rates $\lambda_i, i \in N$, results in a non-cooperative Nash equilibrium strategy.

(b) For non-zero energy costs $\mathcal{E}_t$, $\mathcal{E}_c$ or $\mathcal{E}_f$, the best response of node $i \in N$ given any $\underline{\lambda}_{-i}$ is to set $\lambda_i^j = 0$ for $j \in N - \{i\}$ to minimize energy costs $E_{t,i}(\underline{\lambda})$ and $E_{p,i}(\underline{\lambda})$. If destination group M_i only includes neighbor nodes of node i (i.e. $|R_i| = 0$) and if the throughput reward is greater than total energy cost (i.e $\mathcal{E}_t < |M_i|$), the optimal strategy λ_i^i of node i is $\frac{1}{2}$ for $n = 2$ and $\frac{1}{3}$ for $n \geq 3$ to maximize throughput λ_i and utility u_i. Otherwise, the best strategy of node i is $\lambda_i^j = 0$ for any node j to minimize energy costs and maximize utility u_i such that $\lambda_i = 0$. ☐

Since λ_Σ and $\lambda_{\min}$ are zero in multihop non-cooperative equilibrium, we need to stimulate cooperation of nodes.

6.1. Tit-for-Tat cooperation stimulation

To represent the dependency relationship among nodes in terms of packet forwarding demands, we use a directed dependency graph with edge set D [7]. Each vertex of dependency graph is a node and there is a directed edge from vertex i to vertex j, if there exist a route where i is a relay node and j is a source. Let D_i denote the set of nodes that are in a dependency loop with node i, i.e. $j \in D_i$, if there exists a cycle on dependency graph that includes both nodes i and j. The dependency graph depends on the network topology and communication demands, and is shown in Figure 6 for broadcast communication in tandem networks.

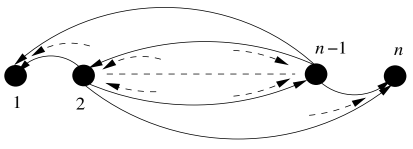

Figure 6. Dependency graph for broadcast communication in tandem networks.

Lemma 1. *For $\mathcal{E}_t > 0$, $\mathcal{E}_f > 0$ or $\mathcal{E}_c > 0$, the unique non-cooperative Nash equilibrium strategy is $\lambda_i^j = 0$, if $j \notin D_i$.*

Proof. Any selfish node i has no incentive to relay packets of node j (i.e. $\lambda_i^j = 0$), if $j \notin D_i$, since the response of node j, i.e. strategies $\underline{\lambda}_j$, cannot affect utility of node i. ☐

The Tit-for-Tat strategy of node i is defined as $\lambda_i^j = \lambda_j^i$ for $j \in D_i$, and $\lambda_i^j = 0$ if $j \notin D_i$, i.e. each node mimics the cooperation level of other nodes, if their strategies depend on each other. It is necessary that each node is able to observe λ_j^i, $j \in D_i$, using a higher level control protocol above network layer. We can further relax the rule as $\lambda_i^j = \lambda_i$ for $j \in D_i$, since $\lambda_i^j > \min_{k \in D_i} \lambda_i^k$ cannot improve throughput but can only increase energy costs, i.e. it is sufficient to observe the value of $\min_{j \in R_i} \lambda_j^i$ or strategies λ_{i-1}^i and λ_{i+1}^i of neighbor nodes, since $\lambda_i^j \geq \lambda_i^k$ for $i < j < k$ or $i > j > k$. The original Tit-for-Tat strategy of $\lambda_i^j = \lambda_j^i$ is reduced to $\lambda_i^j = \min_{k \in D_i} \lambda_k^i$ or simply to $\lambda_i^j = \min_{k \in \{i-1, i+1\} \subseteq D_i} \lambda_k^i$ for any $j \in D_i$, and $\lambda_i^j = 0$ for any $j \notin D_i$. Since strategies of any node depend on strategies of neighbors that can be observed locally, the Tit-for-Tat-based cooperation stimulation can be realized in a distributed manner. We define $D_i^r = D_i \cap N_i^r$ and $D_i^l = D_i \cap N_i^l$, and specify next dependence of non-cooperative Nash equilibrium strategies on dependency graph.

Theorem 3. *The unique non-cooperative Nash equilibrium strategy of node i is $\lambda_i^j = 0$ for all j, if there exists node t such that $(t, i) \in D$ and $t \notin D_i$ or node s such that $(s, t) \in D$, $s \notin D_t$ and $t \in D_i$. Otherwise, the unique equilibrium strategy of node i is $\lambda_i^j = \lambda_i$ for any $j \in i \cup D_i^r \cup D_i^l$.*

Proof. If $(t, i) \in D$ and $t \notin D_i$, node t does not have incentive to forward packets of node i, i.e. $\lambda_t^i = 0$, from Lemma 1. Similarly, if $(s, t) \in D$, $s \notin D_t$ and $t \in D_i$, node s and consequently node t follow the selfish strategies of $\lambda_s^t = 0$ and $\lambda_t^i = 0$, respectively, to minimize energy costs. In both cases, node i reacts with unique strategy of $\lambda_i^j = 0$ for all j to minimize energy costs and maximize u_i.

Otherwise, node i computes its equilibrium strategy λ_i^j by considering that node $j \in D_i$ will respond by setting $\lambda_j^i = \lambda_i^j$. The energy of node i is wasted for relaying packets of node j with rate greater than λ_i, since λ_j^i is set to λ_i^j by node j that employs Tit-for-Tat-based cooperation stimulation and limits λ_i to λ_j^i. Therefore, $\lambda_i^j = \lambda_i$ for any $j \in i \cup D_i^r \cup D_i^l$ to minimize energy costs. □

Theorem 4. *Consider a subset of nodes that are connected to each other on a dependency graph D.*

(a) For network coding, the unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = (\sum_{m=1}^{3} \max_{i:m(i)=m}[1 + \max_{i \in D}(\max(|D_i^r|, |D_i^l|)])^{-1}$ for $j \in D$, if $\max_{i \in D}[\mathcal{E}_t(1 + \max(|D_i^r|, |D_i^l|)) + \mathcal{E}_f(\max(|D_i^r|, |D_i^l|) - \min(|D_i^r|, |D_i^l|)) + \sum_{k \in \{-1,0,1\}} \mathcal{E}_c \min(|D_{i+k}^r|, |D_{i+k}^l|)] < |M_i|$. Otherwise, $\lambda_i^j = 0$, $j \in D$.

(b) For plain routing, the unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = (\sum_{m=1}^{3} \max_{i:m(i)=m}[1 + \max_{i \in D}(|D_i^r| + |D_i^l|])^{-1}$, if $\max_{i \in D}[(\mathcal{E}_t + \mathcal{E}_f)(|D_i^r| + |D_i^l|)] < |M_i| - \mathcal{E}_t$. Otherwise, $\lambda_i^j = 0$, $j \in D$.

Proof. (a) The non-cooperative equilibrium strategy is $\lambda_i^j = \lambda_i$ from Theorem 3. We have a single-variable optimization problem of maximizing u_i subject to achievable throughput conditions (1)-(2) with the resulting common throughput $\lambda_i = \lambda$. For network coding, the utility u_i of node i is $|M_i|\lambda_i - \mathcal{E}_t \lambda_i (1 + \max(|D_i^r|, |D_i^l|)) - \mathcal{E}_f \lambda_i (\max(|D_i^r|, |D_i^l|) - \min(|D_i^r|, |D_i^l|)) - \sum_{k \in \{-1,0,1\}} \mathcal{E}_c \lambda_i \min(|D_{i+k}^r|, |D_{i+k}^l|)$. If $\mathcal{E}_t(1 + \max(|D_i^r|, |D_i^l|)) + \mathcal{E}_f(\max(|D_i^r|, |D_i^l|) - \min(|D_i^r|, |D_i^l|)) + \sum_{k \in \{-1,0,1\}} \mathcal{E}_c \min(|D_{i+k}^r|, |D_{i+k}^l|) < |M_i|$ for all $i \in D$, the utility u_i is an increasing function of $\lambda_i = \lambda$ and maximized by $\lambda_i = (\sum_{m=1}^{3} \max_{i:m(i)=m}[1 + \max_{i \in D}(\max(|D_i^r|, |D_i^l|)])^{-1}$. Otherwise, we have $\lambda_i^j = \lambda_i = 0$ for $j \in D$.

(b) For plain routing, the utility u_i is $|M_i|\lambda_i - \mathcal{E}_t \lambda_i - (\mathcal{E}_t + \mathcal{E}_f)\lambda_i(|D_i^r| + |D_i^l|)$. If $(\mathcal{E}_t + \mathcal{E}_f)(|D_i^r| + |D_i^l|) < |M_i| - \mathcal{E}_t$ for all $i \in D$, the utility u_i of node i is an increasing function of $\lambda_i = \lambda$ and maximized by $\lambda_i = (\sum_{m=1}^{3} \max_{i:m(i)=m}[1 + \max_{i \in D}(|D_i^r| + |D_i^l|])^{-1}$. Otherwise, we have $\lambda_i^j = \lambda_i = 0$ for $j \in D$. □

The non-cooperative Nash equilibrium strategies strongly depend on the dependence graph. As an example, we consider broadcast communication with $M_i = N - \{i\}$:

(a) The unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = 0$ for $i \in N - \{1, n\}$ and $j \in \{1, n\}$ or $\lambda_i^j = 0$ for $i \in \{1, 2\}$ and $j \in N$.

(b) For network coding, the unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = \lambda_i$ for $i \in N - \{1, n\}$ and $j \in N - \{1, n\}$ under condition of Theorem 4-(a) with $|D_i^r| = n - i - 1$ and $|D_i^l| = i - 2$, where $\lambda_i = \frac{1}{2}$ if $n = 2$, $\lambda_i = \frac{1}{3n-8}$ if $n \geq 3$ and $n(\text{mod}3) = 0$ or 1, and $\lambda_i = \frac{1}{3n-7}$ if $n \geq 3$ and $n(\text{mod}3) = 2$.

(c) For plain routing, the unique non-cooperative Nash equilibrium strategies are $\lambda_i^j = \lambda_i$ for $i \in N - \{1, n\}$ and $j \in N - \{1, n\}$ under condition of Theorem 4-(b) with $|D_i^r| = n - i - 1$ and $|D_i^l| = i - 2$, where $\lambda_i = \frac{1}{2}$ if $n = 2$, $\lambda_i = 1$ if $n = 3$, $\lambda_i = \frac{1}{4}$ if $n = 4$ and $\lambda_i = \frac{1}{3(n-2)}$ if $n \geq 5$.

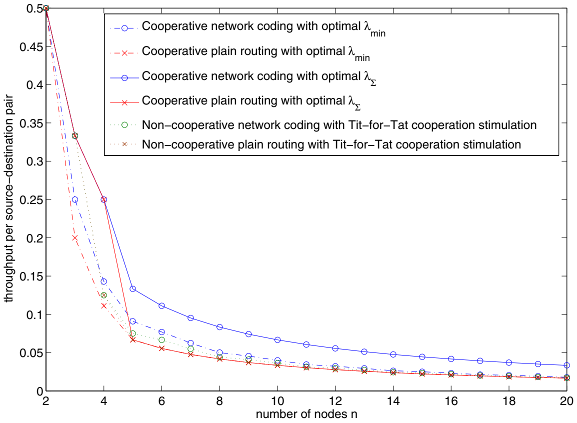

Figure 7. Achievable throughput under cooperative and competitive network operation.

The achievable throughput rates for broadcast communication are depicted in Figure 7 under cooperative and non-cooperative operation. The next question is whether selfish nodes prefer network coding or plain routing. Network coding and plain routing reduce transmission and coding energy costs, respectively. The total energy cost $E_{t,i} + E_{p,i}$ of any node i can be reduced by network coding, if $3\mathcal{E}_c < \mathcal{E}_t + 2\mathcal{E}_f$, and by plain routing otherwise. The Tit-for-Tat-based cooperation stimulation mechanism improves Nash equilibrium. However, the utilities of some nodes (e.g. nodes 1 and n for broadcast communication) are still zero, i.e. $\lambda_{\min} = 0$, and the resulting value of λ_Σ is suboptimal.

6.2. Pricing-based cooperation stimulation

Next, we introduce a pricing-based cooperation mechanism [8] to improve the non-cooperative equilibrium. Each node i pays reward r to each relay node for the throughput unit carried to motivate packet relaying. When a node i computes its equilibrium strategies, node i does not optimize u_i with respect to $\underline{\lambda}_i$ for fixed $\underline{\lambda}_{-i}$, since node i knows a priori that any relay node j is ready to pay $r\lambda_i^j$ to node $i \in R_j$ and node i must pay $r\lambda_j^i$ to node $j \in R_i$. The utility u_i has two additional components of $r\sum_{j \in N_i^r \cup N_i^l} \lambda_i^j$ (the reward for relaying packets of other nodes) and $r\sum_{j \in R_i} \lambda_j^i$ (the cost charged by nodes for relaying packets of node i). For network coding, we assume for simplicity that each coding node pays $\mathcal{E}_c$ to neighbors for any coded packet such that no effective cost is incurred for packet decoding. Next, we evaluate dependence of non-cooperative strategies on r.

Lemma 2. *The unique non-cooperative Nash equilibrium strategy of node i with pricing is* $\lambda_i^j = 0$ *for all* j*, if* $r > \frac{|M_i| - \mathcal{E}_t}{|R_i|}$ *for all* $i \in N$*, or if* $r < \min(\frac{\mathcal{E}_t + 3\mathcal{E}_c}{2}, \mathcal{E}_t + \mathcal{E}_f)$ *for network coding, or if* $r < \mathcal{E}_t + \mathcal{E}_f$ *for plain routing.*

Proof. Node i achieves the total throughput $|M_i|\lambda_i$ at the expense of energy cost $\mathcal{E}_t\lambda_i$ and relaying cost $r|R_i|\lambda_i$. If $r > \frac{|M_i| - \mathcal{E}_t}{|R_i|}$, u_i is decreasing function of λ_i and node i does not have any incentive to generate packets, i.e. $\lambda_i = 0$. Node i cannot unilaterally improve utility u_i for fixed $\underline{\lambda}_{-i}$ and sets $\lambda_i^j = 0$ for any other node j to minimize energy costs. For plain routing, any node i receives reward $r\lambda$ for relaying packets with rate λ at the expense of energy cost $(\mathcal{E}_t + \mathcal{E}_f)\lambda$. For fixed $\underline{\lambda}_{-i}$, relaying packets will decrease utility u_i and therefore $\lambda_i^j = 0$, if $r < \mathcal{E}_t + \mathcal{E}_f$. For network coding, any node i receives reward $2r\lambda$ or $r\lambda$ for relaying packets of two nodes or one node with rate λ at the expense of energy cost $(\mathcal{E}_t + 3\mathcal{E}_c)\lambda$ or $(\mathcal{E}_t + \mathcal{E}_f)\lambda$. For fixed $\underline{\lambda}_{-i}$, relaying packets will decrease utility u_i and therefore the best strategy is $\lambda_i^j = 0$, if $r < \min(\frac{\mathcal{E}_t + 3\mathcal{E}_c}{2}, \mathcal{E}_t + \mathcal{E}_f)$. ☐

Next, we show that non-cooperative operation with pricing can reach the cooperative equilibrium performance.

Theorem 5. *For broadcast communication with network coding, the unique non-cooperative equilibrium strategies with pricing are* $\lambda_1^1 = \lambda_n^n = \frac{1}{3}$ *and* $\lambda_i^1 = \lambda_i^n = \frac{1}{3}$, $\lambda_i^i = 0$ *for* $i \in N - \{1, n\}$*, if* $\frac{n-1-\mathcal{E}_t}{n-2} > r > \max(\min(\frac{n-1+\mathcal{E}_f}{n-2}, \frac{n-1+3\mathcal{E}_c}{n-1}), \min(\frac{\mathcal{E}_t + 3\mathcal{E}_c}{2}, \mathcal{E}_t + \mathcal{E}_f))$*, and maximize* λ_Σ *to the cooperative value of* $\frac{2(n-1)}{3}$.

Proof. For network coding, cooperative equilibrium strategies are $\lambda_1^1 = \lambda_n^n = \frac{1}{3}$ and $\lambda_i^1 = \lambda_i^n = \frac{1}{3}$, $\lambda_i^i = 0$ for $i \in N - \{1, n\}$ and maximize λ_Σ to the value of $\frac{2(n-1)}{3}$. Hence, node $i \in \{1, n\}$ should have the incentive to originate packet traffic. This is possible, if $|M_i| - r|R_i| - \mathcal{E}_t > 0$, where $|M_i| = n - 1$ and $|R_i| = n - 2$, or if $r < \frac{n-1-\mathcal{E}_t}{n-2}$. Also, node $i \in N - \{1, n\}$ with $|R_i| = n - 3$ should prefer relaying packets over originating packets and staying idle, i.e. the utilities of originating packets and staying idle, namely $|M_i| - r|R_i| - \mathcal{E}_t$ and 0, should be smaller than the utility of relaying packets by network coding or plain routing, namely $2r - \mathcal{E}_t - 3\mathcal{E}_c$ or $r - \mathcal{E}_t - \mathcal{E}_f$. ☐

For network coding, it is possible to achieve Pareto optimal operation in terms of λ_Σ. This is not possible for $\mathcal{E}_t > 0$ or $\mathcal{E}_f > 0$, if we restrict nodes to plain routing. However, the cooperative and non-cooperative equilibrium values of $\lambda_{\min}$ are equal for both network coding and plain routing.

Theorem 6. *For broadcast communication, the unique non-cooperative Nash equilibrium strategies with pricing are* $\lambda_i^j = \frac{1}{3n-5}$ *for* $n(mod 3) = 0$ *or* 1*, or* $\frac{1}{3n-4}$ *for* $n(mod 3) = 2$ *under network coding, and* $\lambda_i^j = \frac{1}{2}$ *for* $n = 2$, $\frac{1}{5}$ *for* $n = 3$, $\frac{1}{9}$ *for* $n = 4$ *or* $\frac{1}{3n}$ *for* $n > 4$ *under plain routing, if* $r < \min(\frac{n-1-\mathcal{E}_t}{n-2}, \frac{n-1+3\mathcal{E}_c}{n-1})$ *and* $r < \frac{n-1-\mathcal{E}_t}{n-2}$*, respectively, and maximize* $\lambda_{\min}$ *to the cooperative equilibrium value.*

Proof. For network coding or plain routing, the cooperative equilibrium with optimal $\lambda_{\min}$ is achievable, if every node i has incentive to originate packets, i.e. $\lambda_i^i(|M_i| - r|R_i| - \mathcal{E}_t) > 0$, where $|M_i| = n-1$ and $|R_i| = n-2$ for $i \in \{1, 2\}$ and $|R_i| = n-3$ for $i \in N - \{1, n\}$, i.e. $r < \frac{|M_i| - \mathcal{E}_t}{|R_i|}$. Also, node $i \in N - \{1, n\}$ is required to originate packets rather than relaying packets, if $\lambda_i^i(|M_i| - r|R_i| - \mathcal{E}_t) > \lambda_i^i(2r - \mathcal{E}_t - 3\mathcal{E}_c)$ or $\lambda_i^i(r - \mathcal{E}_t - \mathcal{E}_f)$ for network coding or plain routing, where $|R_i| = n-3$ and $i \in N - \{1, n\}$. Thus, each nodes can individually improve throughput to the maximum value of common cooperative throughput $\lambda_{\min}$. ☐

The next question is whether nodes should prefer network coding or plain routing.

Corollary 1. *For* $r < \min(\frac{n-1-\mathcal{E}_t}{n-2}, \frac{n-1+3\mathcal{E}_c}{n-1})$*, the non-cooperative strategies with pricing use network coding for broadcast communication, if* $3\mathcal{E}_c < \mathcal{E}_t + 2\mathcal{E}_f$.

Proof. If we assume $r < \min(\frac{n-1-\mathcal{E}_t}{n-2}, \frac{n-1+3\mathcal{E}_c}{n-1})$, the non-cooperative equilibrium strategies with pricing can maximize the value of $\lambda_{\min}$ for network coding, as shown in Theorem 6. The utility of node i is $\lambda_i(n - 1 - \mathcal{E}_t \max(|N_i^r|, |N_i^l|) - 3\mathcal{E}_c \min(|N_i^r|, |N_i^l|) -$

$\mathcal{E}_f(\max(|N_i^r|, |N_i^l| - \min(|N_i^r|, |N_i^l|)))$. For plain routing, the utility of node i is $\lambda_i(n - 1 - \mathcal{E}_t(|N_i^r| + |N_i^l|) - \mathcal{E}_f(|N_i^r| + |N_i^l|))$. If $3\mathcal{E}_c < \mathcal{E}_t + 2\mathcal{E}_f$, selfish nodes prefer network coding operations that reduce energy costs and also improve individual throughput, as stated in Theorem 6. $\square$

Distributed implementation is possible through exchange of control packets between neighbor nodes for credit charging provided that nodes agree on common price r. We can also let nodes choose their own packet relaying prices.

6.3. Non-cooperative random access

The possible actions of a node i are waiting (W) or transmitting (T_i^j) a packet of node j, i.e. the action space of node i is $A_i = \{W, \{T_i^j\}_{j \in N_i^r \cup N_i^l}\}$. Let β_i^j denote the probability that transmission of node i carries a packet of node j, where $0 \leq \beta_i^j \leq 1$, $\sum_{j \in i \cup N_i^r \cup N_i^l} \beta_i^j = 1$ for plain routing, or $\beta_i^i + \max(\sum_{j \in N_i^r} \beta_i^j, \sum_{j \in N_i^l} \beta_i^j) = 1$ for network coding. The mixed strategies of nodes are the probability distributions over the actions such that mixed strategy of node i is $\sigma_i = \{p_i, \beta_i^j, j \in N\}$. We define $\underline{\sigma} = \{\sigma_i, i \in N\}$, $\underline{\sigma}_{-i} = \{\sigma_j, j \in N - \{i\}\}$ and $u_i(A|\underline{\sigma}_{-i})$ as the utility of node i provided that node i plays action $A \in A_i$ given $\underline{\sigma}_{-i}$. The mixed Nash equilibrium strategies are $\underline{\sigma}^*$, if for each node i the expected utility $u_i(A|\underline{\sigma}^*_{-i})$ to every action $A \in A_i$ (in support of σ_i^*) is the same for any given $\underline{\sigma}^*_{-i}$.

Nodes do not have incentives to relay packets, i.e. $\beta_i^j = 0, j \in N - \{i\}$, for non-zero energy costs. The Tit-for-Tat-based stimulation mechanism is limited to $\lambda_{\min} = 0$ and cannot optimize λ_Σ. Therefore, we continue with pricing-based cooperation stimulation. The unique non-cooperative Nash equilibrium strategy of each node i is $p_i = 0$, if $r > \frac{|M_i| - \mathcal{E}_t}{|R_i|}$ for all $i \in N$, or if $r < \min(\frac{\mathcal{E}_t + 3\mathcal{E}_c}{2}, \mathcal{E}_t + \mathcal{E}_f)$ under network coding, or if $r < \mathcal{E}_t + \mathcal{E}_f$ under plain routing. The choice of the source packet transmission methods A, B or C is reflected in $\gamma_i(s_i^r, s_i^l)$ for any node i as in section 3.2.

Theorem 7. *Define* $v_{i,j} = s_i^r$ *for* $i < j$, $v_{i,j} = s_i^l$ *for* $i > j$ *and* $v_{i,j} = \gamma_i(s_i^r, s_i^l)$ *for* $i = j$. *For* $j \in N_i^r \cup N_i^l$, *the non-cooperative Nash equilibrium strategies satisfy*
$(|M_i| - r|R_i|)v_{i,i} = \mathcal{E}_t$, $\beta_i^j = \frac{p_j \beta_j^j v_{j,j}}{p_i v_{i,j}}$, $i \in R_j$, *and*
$\max(2r - 3\mathcal{E}_c, r - \mathcal{E}_f)v_{i,j} = \mathcal{E}_t$,
$\beta_i^i = 1 - \max(\sum_{j \in N_i^r} \beta_i^j, \sum_{j \in N_i^l} \beta_i^j)$ *for network coding,*
$(r - \mathcal{E}_f)v_{i,j} = \mathcal{E}_t$, $\beta_i^i = 1 - \sum_{j \in N_i^r \cup N_i^l} \beta_i^j$ *for plain routing.*

Proof. The non-cooperative Nash equilibrium strategies of any node i follow from the equalities of conditional utilities such that $u_i(W|\underline{\sigma}_{-i}) = u_i(T_i^j|\underline{\sigma}_{-i})$ for all $j \in N_i^r \cup N_i^l$, where $u_i(W|\underline{\sigma}_{-i}) = 0$, $u_i(T_i^i|\underline{\sigma}_{-i}) = -\mathcal{E}_t + (|M_i| - r|R_i|)v_{i,i}$ (under the constraint that condition $p_i\beta_i^j v_{i,j} \leq p_j \beta_j^j v_{j,j}$ for $i \in R_j$ is satisfied with equality such that the rate at which packets are relayed is optimized to the rate at which packets are generated) and $u_i(T_i^j|\underline{\sigma}_{-i}) = -\mathcal{E}_t + \max(2r - 3\mathcal{E}_c, r - \mathcal{E}_f)v_{i,j}$ for network coding or $u_i(T_i^j|\underline{\sigma}_{-i}) = -\mathcal{E}_t + (r - \mathcal{E}_f)v_{i,j}$ for plain routing. $\square$

7. Conclusions

In this paper, we considered joint optimization of MAC and network coding in a tandem wireless network with cooperative or competitive nodes. We specified the achievable throughput region for scheduled and random access. Then, we evaluated the cross-layer interactions in cooperative throughput optimization and pointed at trade-offs with energy properties. We also underlined the performance loss of competitive network operation and introduced Tit-for-Tat and pricing-based cooperation stimulation mechanisms to improve the non-cooperative strategies for MAC and network coding. The analysis can be extended to two-dimensional grid networks using orthogonal tandem operations and provides insights for general network topologies.

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