

On the Performance of V-BLAST with Zero-Forcing Successive Interference Cancellation Receiver

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Abstract-The performance analysis of the Zero-Forcing Successive Interference Cancellation (ZF-SIC) detector in Vertical Bell Labs Layered Space-Time (V-BLAST) is generally considered difficult mainly because of the nonlinear error propagation effect. In this paper we first present the nearly exact bit error rate (BER) analysis of ZF-SIC V-BLAST with Binary Phase-Shift Keying (BPSK) modulation in the richly scattered Rayleigh-fading multiple-antenna channel. We derive the closed-form expressions of the error probability of each substream under a given number of transmit antennas. We further give a general recursive procedure to calculate the BER of each substream with arbitrary number of transmit and receive antennas. The procedure we give is easy to follow and is feasible in evaluating the BER performance of V-BLAST with ZF-SIC receiver. It can be further utilized to develop optimal transmission strategy for the open-loop V-BLAST. Our analytical results make excellent matches with the Monte Carlo simulation results.

I INTRODUCTION

Vertical Bell Labs Layered Space-Time (V-BLAST) was proposed [5] to achieve the very high spectral efficiency promised by the multiple-antenna system [1]-[3]. In the original V-BLAST system [5], parallel data streams are simultaneously transmitted through multiple antennas in the same frequency band, and decoded at the receiver with the Zero-Forcing Successive Interference Cancellation (ZF-SIC) detector, which helps to achieve the high spectral efficiency with reasonable decoding complexity. Due to these advantages, ZF-SIC V-BLAST has gained lots of research attention in the past few years [4]-[7].

The performance analysis of V-BLAST using ZF-SIC

detector is in general considered difficult. The difficulty mainly comes from the nonlinear interference cancellation operation, which generates the so-called *error propagation* effect in practical systems and is hard to quantify. Preliminary research in [6] has reported the asymptotic analysis and numerical Monte Carlo simulation results. Although the numerical approach is useful in performance evaluation, the analytical approach provides deep insight and comprehensive understanding of the essential and key points of V-BLAST. Furthermore, analytical results are useful in developing optimal transmission schemes such as power allocation. [7] derives closed-form expressions for signals at each detection step and performs statistical analysis in a Rayleigh-fading channel based on the perfect interference cancellation assumption. [8] breaks this assumption and presents an analysis on the joint error rate and symbol error rate. However the closed-form expression of bit error rate (BER) of each substream is still unknown.

In this paper we are motivated to present a *nearly exact* error probability analysis for the practical ZF-SIC V-BLAST with Binary Phase-Shift Keying (BPSK) modulation in the Rayleigh-fading multiple-antenna channel, in which we will take the effect of error propagation into account. Our analysis will give *closed-form expressions* of BER of each substream when the number of transmit antennas M is fixed. For example we will present the probability of bit error of each substream in the case of $M = 2$. We will also propose a general procedure to calculate the BER of each substream with arbitrary number of transmit and receive antennas. Monte Carlo simulations are used to verify our analytical results. Especially, this analytical result is useful to design the optimal transmission strategy for open-loop V-BLAST, such as the optimal open-loop power allocation.

The rest of this paper is organized as follows. In Section II, we introduce the system model. Brief introduction of ZF-SIC V-BLAST is presented in Section III. Section IV describes the performance analysis and simulation results. Finally,

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Section V contains our conclusions.

II SYSTEM MODEL

We consider a single-user, point to point Rayleigh-fading communication channel with M transmit and N receive antennas. We assume that the channel is flat fading and quasi-static, namely, the channel is constant over a frame, and varies from one frame to another. The fading coefficient \mathbf{h}_j is the complex path gain from transmit antenna j to receive antenna i . We assume that the coefficients are independently complex circular symmetric Gaussian with unit variance, and write $\mathbf{H} = [\mathbf{h}_j] \in \mathbb{C}^{N \times M}$. \mathbf{H} is assumed to be known to the receiver to allow coherent detection, but not at the transmitter. We assume $N \geq M$ so that the channel matrix \mathbf{H} is full column rank with probability one. The total transmission power is assumed to be P_t regardless of M , and is equally allocated on each transmit antenna. The noise is assumed to be additive white Gaussian noise (AWGN) with zero mean and double-sided power spectral density N_0 . The SNR is then given by $SNR = P_t/N_0$.

Specifically, the following discrete-time equivalent model is used:

$$\mathbf{y} = \sqrt{\frac{P_t}{M}} \mathbf{H} \cdot \mathbf{s} + \mathbf{n} \quad (1)$$

where $\mathbf{s} = [s_1, s_2, \dots, s_M]^T$ is a $M \times 1$ vector whose the j -th component represents the signal transmitted from the j -th antenna and belongs to the uncoded normalized (unit average energy) BPSK signal constellation $\{+1, -1\}$. The received signal and noise vector are both $N \times 1$ vectors which are denoted by \mathbf{y} and \mathbf{n} , respectively. We also assume perfect synchronization and timing at the receiver.

The following notations will be used throughout this paper: ' for transpose, \mathbf{I}_n for the $n \times n$ identity matrix, $E[\cdot]$ for expectation, $\text{Cov}(\cdot)$ for covariance, $\text{Prob}\{\cdot\}$ for probability, bold lowercase letters for vector, and bold uppercase letters for matrix. Finally, we shall also find it convenient to partition the channel matrix into its columns as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M]$.

III ZF-SIC RECEIVER FOR V-BLAST

The ZF detector is a linear nulling technique and has the characteristic that its performance approaches that of the minimum mean-squared error (MMSE) detector at high SNRs. SIC is a nonlinear technique first introduced in the theory of multiuser detection (MUD). It exploits the benefit of timing synchronism and uses symbol cancellation to

improve the diversity order [9][10] of the yet to be detected symbols for superior performance. Performance analysis of SIC in MUD has been reported in many literatures such as [11][12]. However those methods may not be applicable in the V-BLAST scenario and a precise analysis of the ZF-SIC over the multiple-antenna channel is still unavailable.

The original structure of the ZF-SIC receiver also comprises the optimal ordering procedure to further enhance the performance. However the optimal ordering will make our analysis much more complicated. In [7] it is also shown that the optimal ordering does not result in increased diversity order, but only in a fixed SNR gain. Thus throughout this paper we only consider the ZF-SIC receiver in a fixed detection order (e.g., detecting according to the order of transmit antenna indexes) instead of the optimal order. It should be noted that the analysis below can be extended in a straightforward way to combine optimal ordering, at the expense of getting more complicated expressions.

We rewrite (1) as

$$\mathbf{y} = \sqrt{\frac{P_t}{M}} \sum_{j=1}^M \mathbf{h}_j s_j + \mathbf{n}$$

The ZF-SIC receiver starts the processing of s_1 and proceeds forwards to s_M . For s_1 , the interfering signal is $\sum_{j=2}^M \mathbf{h}_j s_j$. We choose a weight vector \mathbf{w}_1 according to the ZF criterion to left multiply the received signal \mathbf{y} and get the decision statistic $r_1 = \mathbf{w}_1^* \mathbf{y}$ for s_1 . The weight vector \mathbf{w}_1 actually performs the so-called *interference nulling* procedure [5]. We slice r_1 to obtain \hat{s}_1 , and then the contribution of s_1 to the received signal is totally subtracted under the assumption that $s_1 = \hat{s}_1$. The processing continues for \hat{s}_2 by nulling out interference from substreams 3 to M . Similarly, ZF-SIC proceeds until s_M has been detected.

After subtracting the contribution of s_1, \dots, s_k , we can write the updated received signal as

$$\begin{aligned} \mathbf{y}^{(k)} &= \mathbf{y} - \sum_{j=1}^k \mathbf{h}_j \hat{s}_j \\ &= \underbrace{\sum_{j=k+1}^M \mathbf{h}_j s_j}_{\text{target signal with interference}} + \underbrace{\left(\mathbf{n} + \sum_{j=1}^k \mathbf{h}_j (s_j - \hat{s}_j) \right)}_{\text{equivalent noise}} \end{aligned} \quad (2)$$

It is easy to see from (2) that the updated received signal $\mathbf{y}^{(k)}$ is composed of three parts: the yet to be detected symbols, the noise vector and the potential error propagation

signal. The last two parts make up of the *equivalent noise*. Here we first consider the ideal case where no error propagation is present, and give the performance analysis result of this ideal scenario. The result here will serve as a preliminary for our future analysis in the next section where error propagation is considered.

Assuming perfect feedback, the ZF-SIC creates M independent one-dimensional sub-channels. The j -th sub-channel has the diversity order of $D_j = N - M + j$. [13] calculates the exact probability of bit error on the j -th substream with BPSK modulation as

$$P_{e_j} = \left[\frac{1}{2}(1-\mu) \right]^{D_j} \sum_{t=0}^{D_j-1} \binom{D_j-1+t}{t} \left[\frac{1}{2}(1+\mu) \right]^t \quad (3)$$

where $Q(\bullet)$ denotes the Q-function [13], $\rho_0 = P_t/N_0$, $\mu = \sqrt{\rho/(1+\rho)}$, and $\rho = P_t/(MN_0)$.

IV PERFORMANCE ANALYSIS

In this section we will present the major contribution of our work by giving the nearly exact BER analysis of the ZF-SIC detector with error propagation. Equation (3) reveals the error probability of a substream is totally determined by the *diversity order* D_j and the *substream SNR* ρ . This result will be used extensively in the following analysis. So for the sake of simplicity we define a function

$$\text{Pe}(D, \rho) = \left[\frac{1}{2}(1-\mu) \right]^D \cdot \sum_{t=0}^{D-1} \binom{D-1+t}{t} \left[\frac{1}{2}(1+\mu) \right]^t \quad (4)$$

to represent (3).

We are motivated to calculate the probability of bit error of the k -th substream P_{e_k} , $k=2, \dots, M$. (P_{e_1} can be directly obtained from (3).) First we can write

$$P_{e_k} = \text{Prob}\{s_k \neq \hat{s}_k\} \quad (5)$$

$$= \sum_{i=0}^{k-1} \text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\} \cdot \text{Prob}\{A_{k-1}^i\}$$

where we define the event

$$A_{k-1}^i = \{\text{There are exactly } i \text{ detection errors in } \hat{s}_1 \sim \hat{s}_{k-1}\} \quad (6)$$

It is clear that if we can get the two corresponding parts in (5), we are able to calculate P_{e_k} .

A. Calculation of $\text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\}$

Here we are interested in calculating the conditional probability of error given that $\hat{s}_1 \sim \hat{s}_{k-1}$ have i wrong decisions and $k-1-i$ right decisions. Considering that we do not know exactly which i decisions out of $\hat{s}_1 \sim \hat{s}_{k-1}$ are wrong, we define a map function $g_k(\bullet)$. It could be any injection from $\{1, 2, \dots, i\}$ to $\{1, 2, \dots, k-1\}$. Thus we can write the equivalent noise as

$$\mathbf{n}_k | A_{k-1}^i = \mathbf{n} + \sum_{j=1}^i \mathbf{h}_{g_k(j)} \cdot (s_{g_k(j)} - \hat{s}_{g_k(j)}) \quad (7)$$

The codebook of each transmit antenna is $\{+\sqrt{Pt/M}, -\sqrt{Pt/M}\}$. So $s_{g_k(j)} - \hat{s}_{g_k(j)}$ can only be chosen from $\{+2\sqrt{Pt/M}, -2\sqrt{Pt/M}\}$. It is easy to see that $\mathbf{n}_k | A_{k-1}^i$ is not Gaussian distributed since the event A_{k-1}^i will bring restrictions to \mathbf{n} and $\mathbf{h}_{g_k(j)}$. However we assume here $\mathbf{n}_k | A_{k-1}^i$ is white Gaussian distributed to continue our analysis. This assumption is quite reasonable consolidated by our future simulation results.

We calculate the mean and covariance matrix of $\mathbf{n}_k | A_{k-1}^i$ as

$$\mathbf{E}(\mathbf{n}_k | A_{k-1}^i) = 0, \quad \text{Cov}(\mathbf{n}_k | A_{k-1}^i) = \left[N_0 + \frac{4Pi}{M} \right] \mathbf{I}_{N \times N}$$

As $\mathbf{n}_k | A_{k-1}^i$ is still white Gaussian distributed, we can directly apply (4) to calculate $\text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\}$ as

$$\text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\} = \text{Pe} \left(N - M + k, \frac{P_t}{MN_0 + 4Pi} \right) \quad (8)$$

B. Calculation of $\text{Prob}\{A_{k-1}^i\}$

It is straightforward to get the following results for $k=2, \dots, M$.

When $i=0$, we have

$$\begin{aligned} \text{Prob}\{A_{k-1}^0\} &= \text{Prob}\{\hat{s}_{k-1} = s_{k-1}, A_{k-2}^0\} \\ &= \text{Prob}\{\hat{s}_{k-1} = s_{k-1} | A_{k-2}^0\} \text{Prob}\{A_{k-2}^0\} \\ &= \left[1 - \text{Pe} \left(N - M + k - 1, \frac{P_t}{MN_0} \right) \right] \text{Prob}\{A_{k-2}^0\} \end{aligned} \quad (9)$$

When $i=k-1$, we have

$$\begin{aligned}
\text{Prob}\{A_{k-1}^{k-1}\} &= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1}, A_{k-2}^{k-2}\} \\
&= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^{k-2}\} \cdot \text{Prob}\{A_{k-2}^{k-2}\} \\
&= \text{Pe} \left(N - M + k - 1, \frac{P_i}{MN_0 + 4P_i(k-2)} \right) \cdot \text{Prob}\{A_{k-2}^{k-2}\}
\end{aligned} \tag{10}$$

When $i = 1, \dots, k-1$, we have (11) at the bottom of this page.

We can get $\text{Prob}\{A_0^0\} = 1$ and notice that $\text{Prob}\{A_1^1\} = P_{e_1}$, which can be directly calculated from (3). Thus (9)-(11) are all calculable by recursion from $k = 2$.

C. Calculation of P_{e_k}

Since $\text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\}$ and $\text{Prob}\{A_{k-1}^i\}$ are both calculated, P_{e_k} is directly achieved using (5) for $k = 2, \dots, M$. Obviously, if M is not prior given, we are unable to give the explicit expressions of P_{e_k} because the number of recursion for calculating (9)-(11) cannot be determined. On the contrary, once M is fixed, we are able to get the closed-form expressions of $\text{Prob}\{A_{k-1}^i\}$ and hence get P_{e_k} . For example, we give the closed-form expressions of the BER of each substream when $M = 2$, $\forall N \geq 2$ as follows.

$$\begin{aligned}
P_{e_1} &= \left[\frac{1}{2}(1 - \mu_1) \right]^{N-1} \cdot \sum_{t=0}^{N-2} \binom{N-2+t}{t} \left[\frac{1}{2}(1 + \mu_1) \right]^t \\
P_{e_2} &= (1 - P_{e_1}) \cdot \left[\frac{1}{2}(1 - \mu_1) \right]^N \cdot \sum_{t=0}^{N-1} \binom{N-1+t}{t} \left[\frac{1}{2}(1 + \mu_1) \right]^t \\
&\quad + P_{e_1} \cdot \left[\frac{1}{2}(1 - \mu_2) \right]^N \cdot \sum_{t=0}^{N-1} \binom{N-1+t}{t} \left[\frac{1}{2}(1 + \mu_2) \right]^t
\end{aligned}$$

where $\mu_i = \sqrt{\rho_i / (1 + \rho_i)}$ and $\rho_i = P_i / (2N_0 + 4P_i(i-1))$,

$i = 1, 2$.

Obviously, the closed-form expressions become much more complicated as M increases. So we are motivated to give a general procedure to calculate the nearly exact BER of each substream for arbitrary number of transmit and receive antennas. The whole algorithm comes from (4)-(5), (8)-(11) and can be described compactly through the recursive procedure as follows.

Initialization:

$$\rho(i) = \frac{P_i}{MN_0 + 4P_i(i-1)}, \quad i = 1, \dots, M+1$$

$$P_{e_1} = \text{Pe}(N - M + 1, \rho(1))$$

$$\text{Prob}\{A_0^0\} = 1$$

Recursion:

for $k=2:M$

$$\text{Prob}\{A_{k-1}^0\} = [1 - \text{Pe}(N - M + k - 1, \rho(1))] \cdot \text{Prob}\{A_{k-2}^0\}$$

$$\text{Prob}\{A_{k-1}^{k-1}\} = \text{Pe}(N - M + k - 1, \rho(k-1)) \cdot \text{Prob}\{A_{k-2}^{k-2}\}$$

if $k > 2$

for $i=1:k-2$

$$\begin{aligned}
\text{Prob}\{A_{k-1}^i\} &= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^{i-1}\} \cdot \text{Prob}\{A_{k-2}^{i-1}\} \\
&\quad + [1 - \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^i\}] \cdot \text{Prob}\{A_{k-2}^i\}
\end{aligned}$$

end

end

for $i=0:k-1$

$$\text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\} = \text{Pe}(N - M + k, \rho(i+1))$$

end

$$P_{e_k} = \sum_{i=0}^{k-1} \text{Prob}\{s_k \neq \hat{s}_k | A_{k-1}^i\} \cdot \text{Prob}\{A_{k-1}^i\}$$

end

Function $\text{Pe}(D, \rho)$ is defined in (4).

$$\begin{aligned}
\text{Prob}\{A_{k-1}^i\} &= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1}, A_{k-2}^{i-1}\} + \text{Prob}\{s_{k-1} = \hat{s}_{k-1}, A_{k-2}^i\} \\
&= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^{i-1}\} \cdot \text{Prob}\{A_{k-2}^{i-1}\} + \text{Prob}\{s_{k-1} = \hat{s}_{k-1} | A_{k-2}^i\} \cdot \text{Prob}\{A_{k-2}^i\} \\
&= \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^{i-1}\} \cdot \text{Prob}\{A_{k-2}^{i-1}\} + [1 - \text{Prob}\{s_{k-1} \neq \hat{s}_{k-1} | A_{k-2}^i\}] \cdot \text{Prob}\{A_{k-2}^i\}
\end{aligned} \tag{11}$$

We resort to Monte Carlo simulations to verify our analysis result. We consider an uncoded V-BLAST system with 4 transmit antennas and 4 receive antennas. BPSK modulation is adopted at the transmitter and ZF-SIC with index-order detection is performed at the receiver. Uncoded BER in simulation is obtained by averaging over large volumes of channel realizations. Fig. 1 gives the BER performance obtained from both the simulation and the analysis. It is clear that the Monte Carlo simulation makes a nearly perfect match to our analysis result, which demonstrates the validity of the calculating method we have proposed. To the best of our knowledge, there is no such accurate calculation reported.

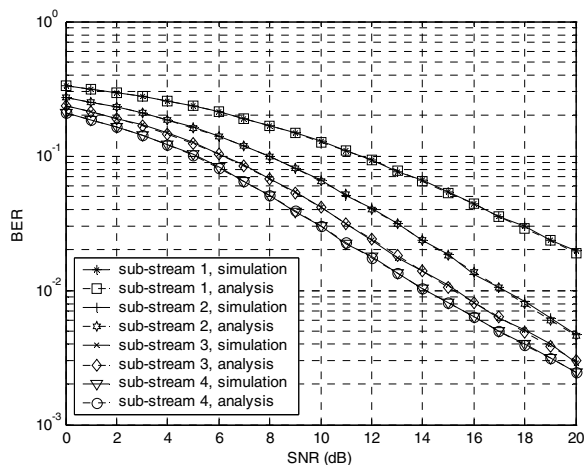


Fig. 1. Comparison between simulation results and our analysis. $M=N=4$, BPSK modulation, uncoded ZF-SIC V-BLAST.

V CONCLUSIONS

We have solved the problem of calculating the nearly exact BER of ZF-SIC V-BLAST with BPSK modulation in a richly scattered Rayleigh-fading multiple-antenna channel. Given the number of transmit antennas M , we can give the closed-form expressions of the average probability of bit error of each substream.

We also proposed a general recursive procedure to calculate the BER of each substream with arbitrary number of transmit and receive antennas. The procedure is easy to follow and is feasible in evaluating the BER performance of V-BLAST with ZF-SIC receiver. Simulation results have demonstrated the validity of our analysis.

Our method can be extended to the high-order modulation

schemes by making some approximations. The analytical result can be further utilized to develop optimal transmission strategy for the open-loop V-BLAST. We will treat these topics in a future work [14].

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