Problem Definitions, Reductions & Expressing Them as Programs

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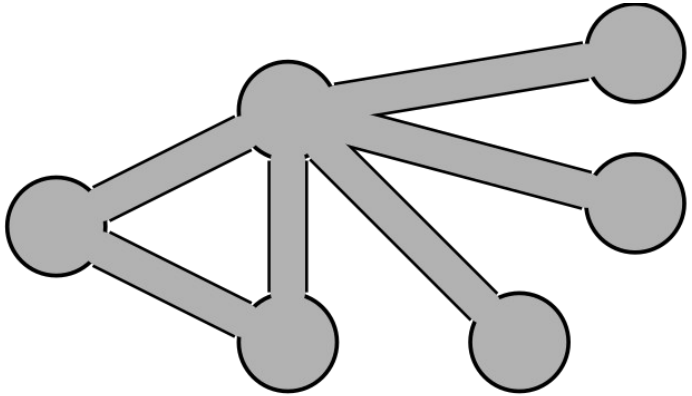
CS396 Spring 2023
Northwestern

Plan of the week

- NP Problem & Reduction -- Monday
- **Examples, Reduction in Karp (Today)**
- Lab, Assignment 4 -- Friday

Review – NP problem

Can we cover all edges by selecting only **2** vertices?



VERTEX-COVER

Review – NP problem

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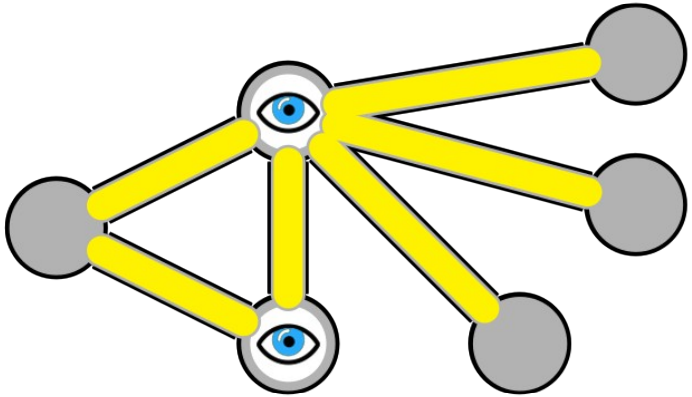

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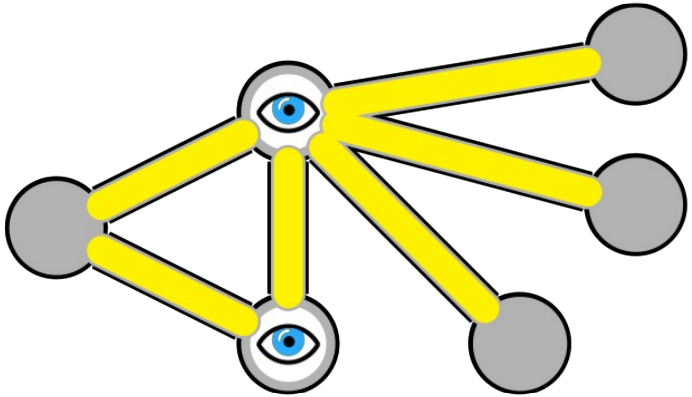
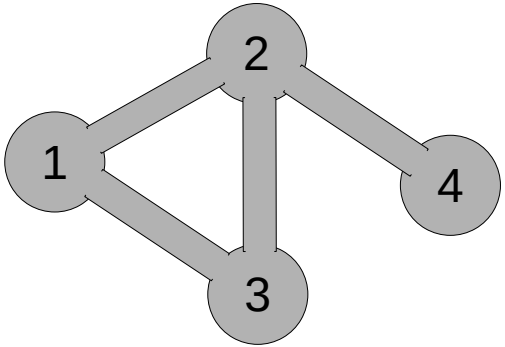


VERTEX-COVER

Yes-instance has easy to check certificates

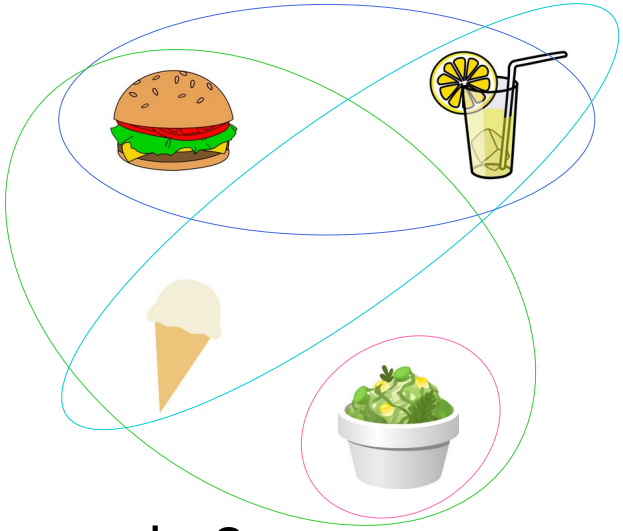
Review – Reduction and Justification



k=2

VERTEX-COVER

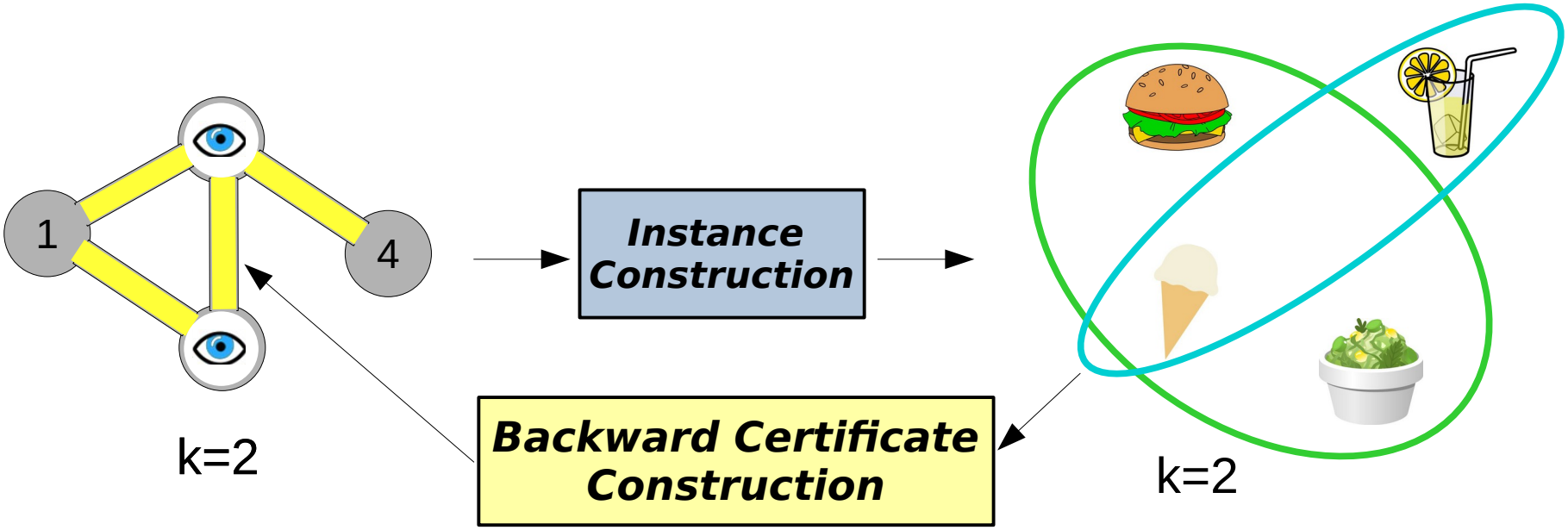
Instance Construction



k=2

SET-COVER

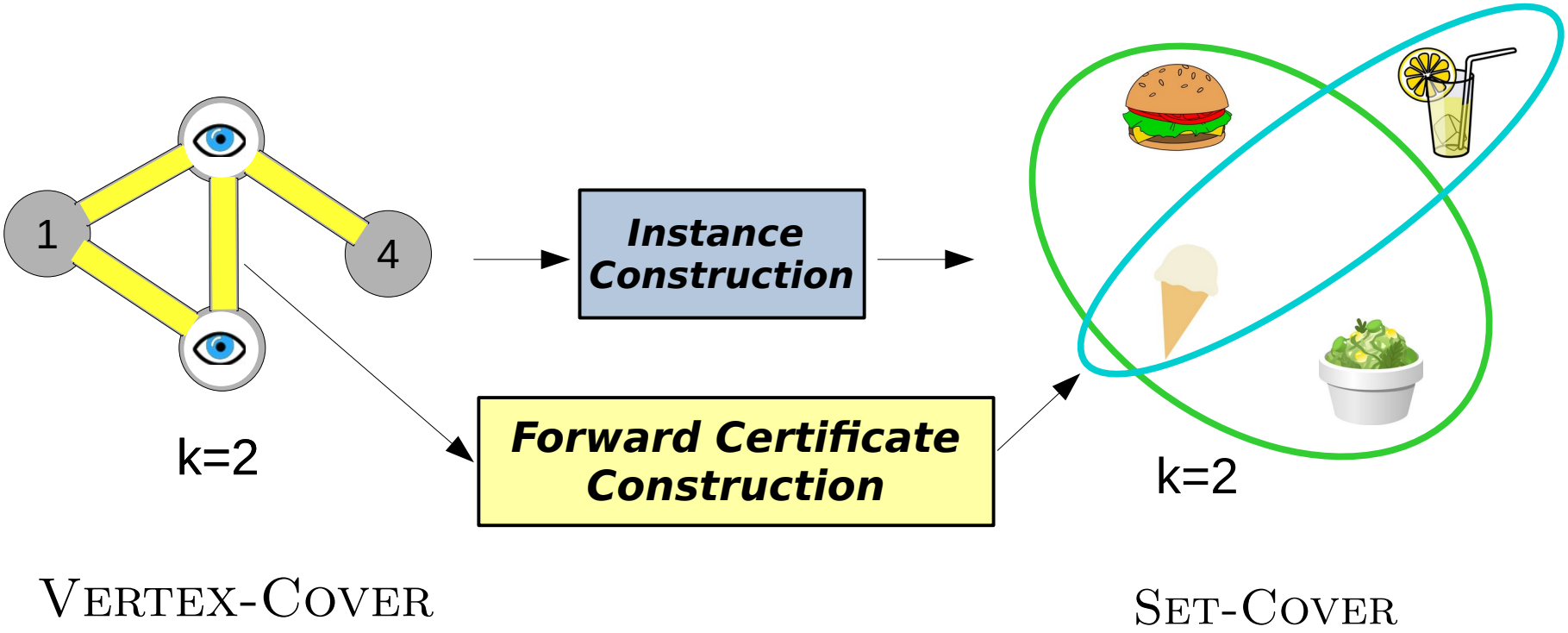
Backward Certificate Construction



VERTEX-COVER

SET-COVER

Forward Certificate Construction



VERTEX-COVER Instance and Certificate

Instance:

Certificate:

Vertex-Cover Instance and Certificate

Instance: graph G and natural k

Certificate:

Vertex-Cover Instance and Certificate

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Certificate: subset of vertices of G

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Assertion for valid certificate C of (G,k):

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Exists v in C s.t. v in endpoint of e

VERTEX-COVER Instance and Certificate

Instance: graph G and natural k

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Assertion for valid certificate C of (G,k):

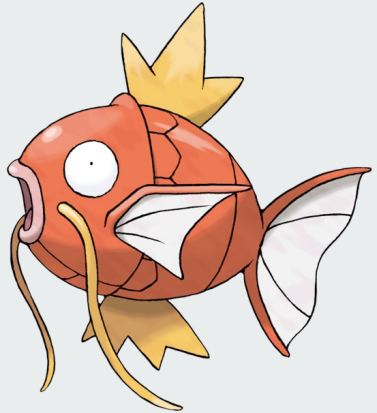
Forall e in edges of G:

Exists v in C s.t. v in endpoint of e

and

Size of C <= k

#lang karp



#lang karp

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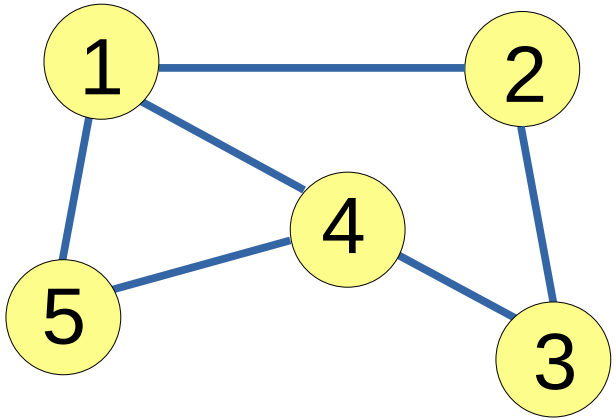
(named after Richard M. Karp)

Reduction Example

H = 3-SAT **N =** INDEPENDENT-SET

Independent-Set

Exists a set of k vertices s.t. no two are neighbors of each other?

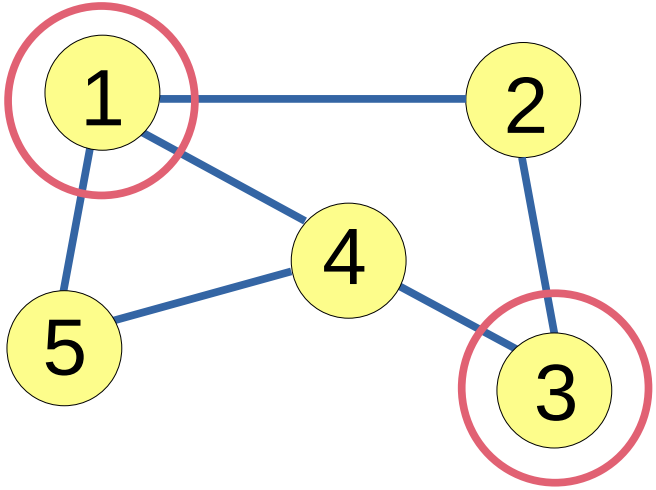


k=2

Instance: a graph G and a threshold number k

Independent-Set

Exists a set of k vertices s.t. no two are neighbors of each other?



k=2

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

Independent-Set

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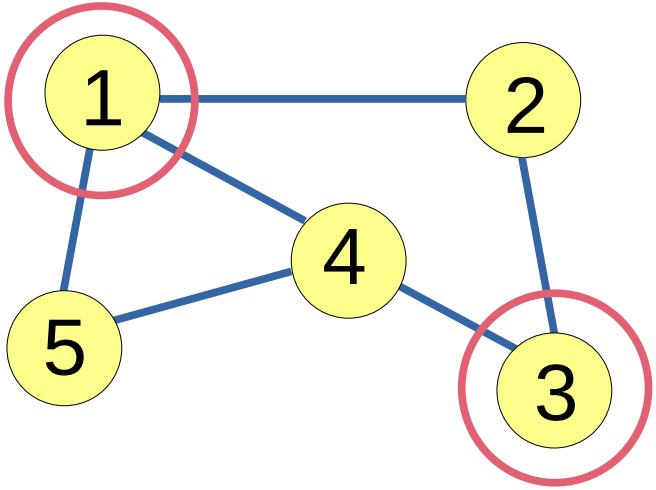

Valid Certificate

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

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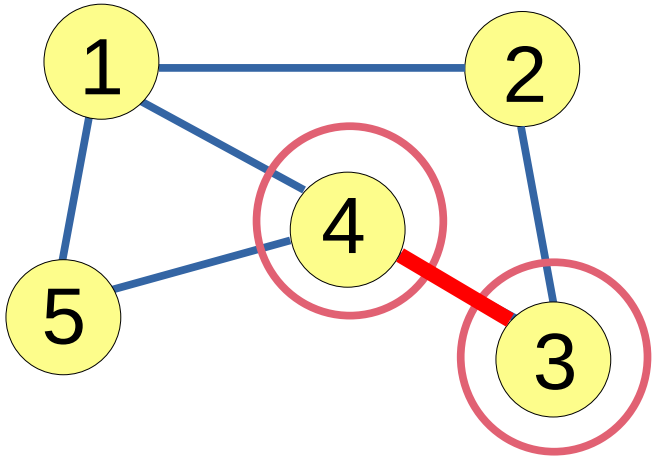

Invalid Certificate

Instance: a graph G and a threshold number k

Certificate: a subset of the vertices of G

Independent-Set Instance and Certificate

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Instance: graph G and natural k

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Forall e in edges of G:

Independent-Set Instance and Certificate

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Assertion for valid certificate C of (G,k):

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Not (And one vertex of e in C

the other vertex of e in C)

Independent-Set Instance and Certificate

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Size of C >= k